

Diploma Mathematics formulae(20SC01T)

Co - ordinate geometry

- Slope of a straight line $m = \tan\theta$
- Slope of line joining two points $m = \frac{y_2 - y_1}{x_2 - x_1}$
- General form of equation of straight line
 $ax + by + C = 0$
- Slope of a straight line $= -\frac{a}{b}$
 $X - \text{intercept} = -\frac{b}{a}$
 $Y - \text{intercept} = -\frac{c}{a}$
- Slope intercept form $y = mx + C$
- Two point form of a straight line
 $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$
- Slope point form of a straight line
 $y - y_1 = m(x - x_1)$
- Intercept form of the straight line
 $\frac{x}{a} + \frac{y}{b} = 1$
- Equation of the straight line which is parallel to line $ax + by + c = 0$ and passing through the point (x_1, y_1) is
 $ax_1 + by_1 + K = 0$
- Equation of the straight line which is perpendicular to the line
 $ax + by + c = 0$ and passing through the point (x_1, y_1) is $bx_1 - ay_1 + k = 0$
- Midpoint of the line formed by the points (x_1, y_1) and (x_2, y_2) is given by

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Trigonometric formulae

Radian to degree conversion and vice versa

$x \text{ radian} = x \frac{180}{\pi} \text{ degree}$		$x \text{ degree} = \frac{\pi}{180} \times x \text{ radians}$	
$\frac{\pi}{2}$	$\frac{\pi}{2} \times \frac{180}{\pi} = 90^\circ$	90°	$90^\circ \times \frac{\pi}{180} = \frac{\pi}{2}$
$\frac{\pi}{4}$	45°	45°	$\frac{\pi}{4}$
$\frac{\pi}{12}$	15°	15°	$\frac{\pi}{12}$
$\frac{5\pi}{12}$	75°	75°	$\frac{5\pi}{12}$

T - Values of standard angles

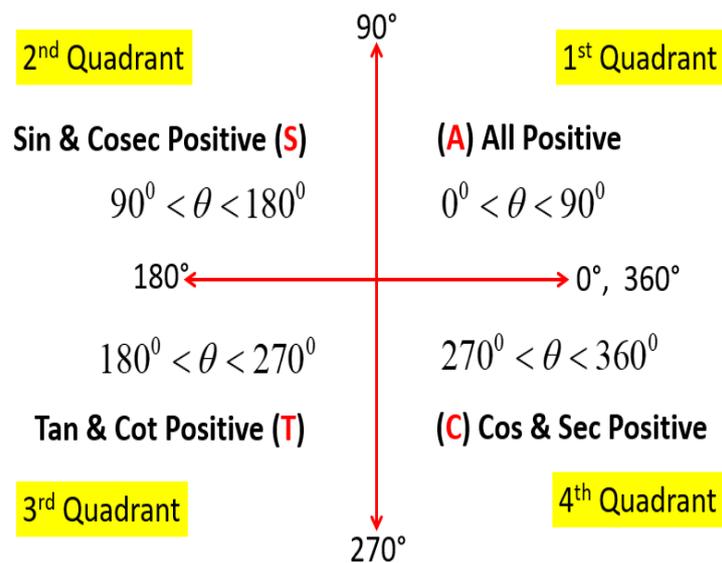
	0°	30°	45°	60°	90°	180°	270°	360°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	1	0	1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	$-\infty$	0
cot	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	∞	0	∞
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞	1	∞	1
cosec	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	∞	-1	∞

Trigonometric ratios of Allied angles (without proof)

Trigonometric ratios of allied angles, when the sum or difference of two angles is either zero or a multiple of 90° . For example 30° and 60° are allied angles because their sum is 90° .

The angles $-\theta, 90^\circ \pm \theta, 180^\circ \pm \theta, 360^\circ \pm \theta$ etc. are angles allied to the angle θ , if θ is measured in degrees. However, if θ is measured in radians, then the angles allied to θ are $-\theta, \frac{\pi}{2} \pm \theta, \pi \pm \theta, 2\pi \pm \theta$ etc.

Using trigonometric ratios of allied angles we can find trigonometric ratios of angles of any magnitude.



Note:

Easy steps to find the allied angle

Step 1(Negativiting)

If it's a t ratio of negative angle change it to positive by using the following table

$\sin(-\theta) = -\sin\theta$	$\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$
$\cos(-\theta) = +\cos\theta$	$\sec(-\theta) = \sec\theta$
$\tan(-\theta) = -\tan\theta$	$\cot(-\theta) = -\cot\theta$

Step 2(Splitting)

Express the given angle as $90 \pm \theta$, $180 \pm \theta$, $270 \pm \theta$ and $360 \pm \theta$, Check which group does it belongs to (we have divided into two groups 1. Odd group($90 \pm \theta$ and $270 \pm \theta$) and 2. Even group ($180 \pm \theta$, $360 \pm \theta$)

Step 3(Grouping)

If it belongs to even group T function doesn't changes

If it belongs to odd group T ratio changes ($\sin\theta \leftrightarrow \cos\theta$, $\operatorname{cosec}\theta \leftrightarrow \sec\theta$, $\tan\theta \leftrightarrow \cot\theta$)

Step 4(Quadranting)

To assign the sign for the obtained value ,follow the ASTC rule

Compound Angles Formulae:

Addition Formulae:

- $\sin(A + B) = \sin A \cos B + \cos A \sin B$
- $\cos(A + B) = \cos A \cos B - \sin A \sin B$
- $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

Subtraction Formulae:

- $\sin(A - B) = \sin A \cos B - \cos A \sin B$
- $\cos(A - B) = \cos A \cos B + \sin A \sin B$
- $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

Multiple Angles

- $\sin 2A = 2 \sin A \cos A$
- $\sin 2A = \frac{2 \tan A}{1 + \tan^2 A}$
- $\cos 2A = \cos^2 A - \sin^2 A$
- $\cos 2A = 2 \cos^2 A - 1$
- $\cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$
- $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$

Angle/ Function	$-\theta$	$90^\circ - \theta$ OR $\frac{\pi}{2} - \theta$	$90^\circ + \theta$ OR $\frac{\pi}{2} + \theta$	$180^\circ - \theta$ OR $\pi - \theta$	$180^\circ + \theta$ OR $\pi + \theta$	$270^\circ - \theta$ OR $\frac{3\pi}{2} - \theta$	$270^\circ + \theta$ OR $\frac{3\pi}{2} + \theta$	$360^\circ - \theta$ OR $2\pi - \theta$	$360^\circ + \theta$ OR $2\pi + \theta$
cosecant	$-\operatorname{cosec}\theta$	$\sec\theta$	$\sec\theta$	$\operatorname{cosec}\theta$	$-\operatorname{cosec}\theta$	$-\operatorname{cosec}\theta$	$\operatorname{cosec}\theta$	$-\operatorname{cosec}\theta$	$\operatorname{cosec}\theta$
secant	$\sec\theta$	$\operatorname{cosec}\theta$	$-\operatorname{cosec}\theta$	$-\sec\theta$	$-\sec\theta$	$\operatorname{cosec}\theta$	$\operatorname{cosec}\theta$	$\sec\theta$	$\sec\theta$
cotangent	$-\cot\theta$	$\tan\theta$	$-\tan\theta$	$-\cot\theta$	$\cot\theta$	$\tan\theta$	$-\tan\theta$	$-\cot\theta$	$\cot\theta$
tangent	$-\tan\theta$	$\cot\theta$	$-\cot\theta$	$-\tan\theta$	$\tan\theta$	$\cot\theta$	$-\cot\theta$	$-\tan\theta$	$\tan\theta$
Cosine	$\cos\theta$	$\sin\theta$	$-\sin\theta$	$-\cos\theta$	$-\cos\theta$	$-\sin\theta$	$\sin\theta$	$\cos\theta$	$\cos\theta$
Sine	$-\sin\theta$	$\cos\theta$	$\cos\theta$	$\sin\theta$	$-\sin\theta$	$-\cos\theta$	$-\cos\theta$	$-\sin\theta$	$\sin\theta$

Table : 4.1

- $\sin 3A = 3 \sin A - 4 \sin^3 A$
- $\cos 3A = 4 \cos^3 A - 3 \cos A$
- $\tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$

Transformation of sum or difference into product

- $\sin C + \sin D = 2 \sin \left(\frac{C+D}{2} \right) \cos \left(\frac{C-D}{2} \right)$
- $\sin C - \sin D = 2 \cos \left(\frac{C+D}{2} \right) \sin \left(\frac{C-D}{2} \right)$
- $\cos C + \cos D = 2 \cos \left(\frac{C+D}{2} \right) \cos \left(\frac{C-D}{2} \right)$
- $\cos C - \cos D = -2 \sin \left(\frac{C+D}{2} \right) \sin \left(\frac{C-D}{2} \right)$

Transformation of product into sum or difference

1. $\sin A \cos B = \frac{1}{2} [\sin(A + B) + \sin(A - B)]$
2. $\cos A \sin B = \frac{1}{2} [\sin(A + B) - \sin(A - B)]$
3. $\cos A \cos B = \frac{1}{2} [\cos(A + B) + \cos(A - B)]$
4. $\sin A \sin B = \frac{1}{2} [\cos(A - B) - \cos(A + B)]$

DIFFERENTIAL CALCULUS

Derivatives of algebraic functions

1. $\frac{d}{dx}(x) = 1$
2. $\frac{d}{dx}(x^2) = 2x$
3. $\frac{d}{dx}(x^3) = 3x^2$
4. $\frac{d}{dx}(x^n) = nx^{n-1}$ where $n \in \mathbb{R}$
5. $\frac{d}{dx}\left(\frac{1}{x^n}\right) = -\frac{n}{x^{n+1}}$ where $n \in \mathbb{R}$
6. $\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$
7. $\frac{d}{dx}\left(\frac{1}{x^2}\right) = -\frac{1}{x^3}$
8. $\frac{d}{dx}(k) = 0$ where k is constant
9. $\frac{d}{dx}(1) = 0$
10. $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$
11. $\frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right) = -\frac{1}{2x\sqrt{x}}$
12. $\frac{d}{dx}(ku) = k\left(\frac{d}{dx}\right)$

Derivatives of trigonometric functions

13. $\frac{d}{dx}(\sin x) = \cos x$
14. $\frac{d}{dx}(\cos x) = -\sin x$
15. $\frac{d}{dx}(\tan x) = \sec^2 x$
16. $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$
17. $\frac{d}{dx}(\sec x) = \sec x \tan x$
18. $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

Derivatives of Inverse trigonometric functions

19. $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$
20. $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$
21. $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
22. $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$

$$23. \frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$$

$$24. \frac{d}{dx}(\operatorname{cosec}^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$$

Derivatives of exponential functions

$$25. \frac{d}{dx}(a^x) = a^x \log a$$

$$26. \frac{d}{dx}(e^x) = e^x$$

Derivatives of logarithmic functions

$$27. \frac{d}{dx}(\log x) = \frac{1}{x}$$

Sum rule

Note: u, v, w are the functions of 'x'

$$28. \frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$$

$$29. \frac{d}{dx}(u + v + w) = \frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx}$$

Product rule

$$30. \frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$31. \frac{d}{dx}(uvw) = uv \frac{dw}{dx} + vw \frac{du}{dx} + wu \frac{dv}{dx}$$

Quotient rule

$$32. \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu^1 - uv^1}{v^2} \text{ or}$$

$$\frac{d}{dx}\left(\frac{Nr}{Dr}\right) = \frac{(Dr \frac{d(Nr)}{dx} - Nr \frac{d(Dr)}{dx})}{Dr^2}$$

where $Nr \rightarrow$ numerator, $Dr \rightarrow$ denominator

33. Slope of a tangent to the curve $y = f(x)$ at the point $p(x_1, y_1)$ is

$$m = \left(\frac{dy}{dx}\right)$$

34. Equation of tangent to the curve

$y = f(x)$ at the point $p(x_1, y_1)$ is

$y - y_1 = m(x - x_1)$ where m is slope of a tangent

35. Slope of a normal to the curve $y = f(x)$

at the point $p(x_1, y_1)$ is $= -\frac{1}{m} = -\frac{1}{\left(\frac{dy}{dx}\right)}$

36. Equation of normal to the curve $y = f(x)$ at the point $p(x_1, y_1)$

$$y - y_1 = -\frac{1}{m}(x - x_1)$$

INTEGRAL CALCULUS

$$1. \int 0 dx = C$$

$$2. \int 1 dx = x + C$$

$$3. \int k dx = kx + C$$

$$4. \int x^n dx = \frac{x^{n+1}}{n+1} + C; n \neq -1$$

$$5. \int \frac{1}{x^n} dx = -\frac{1}{(n-1)x^{n-1}} + c; n \neq 1$$

$$6. \int \frac{1}{x} dx = \ln |x| + C$$

$$7. \int \frac{1}{x^2} dx = -\frac{1}{x} + C$$

$$8. \int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + C$$

$$9. \int \sqrt{x} dx = \frac{2}{3}x^{\frac{3}{2}} + C$$

$$10. \int a^x dx = \frac{a^x}{\log a} + C; a > 0, a \neq 1$$

$$11. \int e^x dx = e^x + C$$

$$12. \int \sin x dx = -\cos x + C$$

$$13. \int \cos x dx = \sin x + C$$

$$14. \int \tan x dx = \log \sec x + c$$

$$15. \int \cot x dx = \log \cos x + c$$

$$16. \int \sec x dx = \log(\sec x + \tan x) + c$$

$$17. \int \operatorname{cosec} x dx = \log(\operatorname{cosec} x - \cot x) + c$$

$$18. \int \sec^2 x dx = \tan x + C$$

$$19. \int \operatorname{cosec}^2 x dx = -\cot x + C$$

$$20. \int \sec x (\tan x) dx = \sec x + C$$

$$21. \int \operatorname{cosec} x (\cot x) dx = -\operatorname{cosec} x + C$$

$$22. \int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$$

$$23. \int \frac{1}{1+x^2} dx = \tan^{-1} x + C$$

$$24. \int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + C$$

$$25. \int -\frac{1}{\sqrt{1-x^2}} dx = \cos^{-1} x + C$$

$$26. \int -\frac{1}{1+x^2} dx = \cot^{-1} x + C$$

$$27. \int -\frac{1}{x\sqrt{x^2-1}} dx = \operatorname{cosec}^{-1} x + C$$

$$28. \int \sin(ax+b) dx = -\frac{\cos(ax+b)}{a} + C$$

$$29. \int \cos(ax+b) dx = \frac{\sin(ax+b)}{a} + C$$

$$30. \int e^{ax+b} dx = \frac{e^{ax+b}}{a} + C$$

$$31. \int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$$

$$32. \int \frac{1}{ax+b} dx = \frac{\log(ax+b)}{a} + C$$

Sum rule

$$33. \int (f(x) + g(x)) dx = \int f(x) dx + \int g(x) dx$$

Difference rule

$$34. \int (f(x) - g(x)) dx = \int f(x) dx - \int g(x) dx$$

Product rule

$$35. \int uv dx = u \int v dx - \int (u \frac{dv}{dx}) dx$$

where 'u' and 'v' are the functions of x

Definite integral

36. If $\int f(x) dx = \phi(x)$ then

$$\int_a^b f(x) dx = [\phi(x)](b-a) = \phi(b) - \phi(a)$$

37. The area bounded by the curve $y = f(x)$, x-axis between the co-ordinates $x = a$ and $x = b$ is

$$\text{Area} = \int_a^b y dx = \int_a^b f(x) dx$$

38. The area bounded by the curve $y = f(x)$, y-axis between the co-ordinates $y = a$ and $y = b$ is

$$\text{Area} = \int_a^b x dy = \int_a^b g(y) dy$$

39. Volume of solid generated about x-axis is:

$$\text{Volume} = \pi \int_a^b y^2 dx$$

40. Volume of solid generated about y-axis is:

$$\text{Volume} = \pi \int_a^b x^2 dy$$