Makeup Examination – Sept. 2023
I/II//IV Semester Diploma Examination

ENGINEERING MATHEMATICS (20SC01T)

(Exam Date / Time: 23rd Sep. 2023 / 2.00 PM)

Time: 3 hours **Max Marks: 100**

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1. (a) If $A=\begin{bmatrix}2 & 1\\ 3 & 4\end{bmatrix}$ *and* $B=\begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix}$, then find $2A+3B$.

OR

$$2A+3B = 2\begin{bmatrix}2 & 1\\ 3 & 4\end{bmatrix} + 3\begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix}$$

$$= \begin{bmatrix}4 & 2\\ 6 & 8\end{bmatrix} + \begin{bmatrix}3 & 12\\ 6 & 9\end{bmatrix}$$

$$= \begin{bmatrix}7 & 14\\ 12 & 17\end{bmatrix}$$

OR

If $A = \begin{bmatrix} 3 & 2 \\ 2 & 0 \end{bmatrix}$ then find $A + A^T$ matrix .

Sol^n

$$A + A^T = \begin{bmatrix} 3 & 2 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 2 \\ 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 4 \\ 4 & 0 \end{bmatrix}$$

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Find the characteristics roots of the matrix $A = \begin{bmatrix} 5 & 2 \\ 4 & 3 \end{bmatrix}$

Sol^n

$$|A - \lambda I| = 0$$

$$\lambda^2 - (Tr\,A)\lambda + \Delta = 0$$

$$Tr\,A = 8, \quad \Delta = \begin{vmatrix} 5 & 2 \\ 4 & 3 \end{vmatrix} = 15 - 8 = 7$$

$$\boxed{\lambda^2 - 8\lambda + 7 = 0}$$

$$\boxed{x_1 = 7}, \quad \boxed{x_2 = 1}$$

$+7$

$-7 \quad -1$

$= -8$

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OR

Find the inverse of the matrix $A = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$

$$A^{-1} = \frac{\text{adj} A}{|A|}$$

1m

$$\text{adj} A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix} \quad \&$$

2m

$$|A| = \begin{vmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{vmatrix} = \cos^2 x + \sin^2 x = 1$$

2m

$$\boxed{A^{-1} = \frac{\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}}{1}}$$

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Find the inverse of the matrix

(c) Find the adjoint of the matrix $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

OR

$\Rightarrow$ $\underline{\text{Soln}}$

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$\Rightarrow \boxed{\text{adj}A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}} \checkmark$$

Solve the equations 2x+y=1; 3x+2y=1 by using Cramer's rule.

Sol^n $2x+y=1$

$3x+2y=1$

$\Rightarrow \quad \Delta = \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = 4-3 = 1$

$\Delta_x = \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 2-1 = 1$

$\Delta_y = \begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} = 2-3 = -1$

$\boxed{x = \frac{\Delta x}{\Delta} = \frac{1}{1} = 1}$, $\boxed{y = \frac{\Delta y}{\Delta} = \frac{-1}{1} = -1}$

LHS: $2x+y = 2(1)-1 = 2-1 = 1 =$ RHS

$\boxed{x = 1}$, $\boxed{y = -1}$

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(d) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$, find AB matrix and also find $(AB)^T$ matrix. 5

OR

Sol^n

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2+6 & 1+8 \\ 6+12 & 3+16 \end{bmatrix} = \begin{bmatrix} 8 & 9 \\ 18 & 19 \end{bmatrix}$$

$$AB = \begin{bmatrix} 8 & 9 \\ 18 & 19 \end{bmatrix}$$

$$\boxed{(AB)^T = \begin{bmatrix} 8 & 18 \\ 9 & 19 \end{bmatrix}}$$

OR

If $\begin{vmatrix} x & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 0 & x \end{vmatrix}$ find the value of x.

Sol^n

$$\begin{vmatrix} x & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 0 & x \end{vmatrix}$$

$$\Rightarrow \quad 4x - 6 = 3x - 0$$

$$\Rightarrow \quad 4x - 3x = 6$$

$$\Rightarrow \quad \boxed{x = 6}$$

Section-II

2. (a) Find the equation of a straight line with slope 5 and y-intercept 3. 4

OR

Write the standard form of equation of straight line with

Sol^n

$m = 5$, $c = 3$

$y = mx + c$

$y = 5x + 3$

$\Rightarrow 0 = 5x - y + 3$

$\Rightarrow \boxed{5x - y + 3 = 0}$

a) One point (x_1, y_1) having slope m.

b) Two points (x_1, y_1) and (x_2, y_2).

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Solⁿ

a) $$\boxed{y - y_1 = m(x - x_1)}$$

b) $$\boxed{\frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1}}$$

(b) Find the equation straight line passing through two points (3,4) and (5,6) 6

OR

Find equation of straight line passing through the point (1,2) which makes an angle 45^o With positive direction of x- axis.

Sol^n

$\Rightarrow \quad \theta = 45^\circ$

$\Rightarrow \quad m = \tan\theta = \tan 45^\circ = 1$

$(x_1, y_1) = (1, 2)$

$\Rightarrow \quad y - y_1 = m(x - x_1)$

$\Rightarrow \quad y - 2 = 1(x - 1)$

$\Rightarrow \quad y - 2 = x - 1$

$\Rightarrow \quad 0 = x - y - 1 + 2$

$$\boxed{0 = x - y + 1}$$

(c) Find the acute angle between the lines x-2y+1=0 and 2x+6y-5=0. 5

OR

Sol^n

$x - 2y + 1 = 0$

$a = 1,\ b = -2$

$m_1 = \frac{-a}{b} = \frac{-1}{-2} = \frac{1}{2}$

$2x + 6y - 5 = 0$

$m_2 = -\frac{\cancel{2}}{\cancel{6}\,3}$

$= -\frac{1}{3}$

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$$

$$= \left|\frac{-\frac{1}{3} - \frac{1}{2}}{1 + \left(\frac{1}{2}\right)\left(-\frac{1}{3}\right)}\right| = \left|\frac{-\cancel{\frac{5}{6}}}{\cancel{\frac{5}{6}}}\right|$$

$= 1$

$\Rightarrow \tan\theta = 1$

$\Rightarrow \boxed{\theta = 45^\circ}$

Prove that the given straight lines 3x-4y-7=0 and 9x-12y-11=0 are parallel.

Soln

$$3x - 4y - 7 = 0$$

$$m_1 = \frac{\cancel{-}3}{\cancel{-}4} = \frac{3}{4}$$

& $9x - 12y - 11 = 0$

$$m_2 = \frac{\cancel{-}\cancel{9}^{3}}{\cancel{-}\cancel{12}_{4}} = \frac{3}{4}$$

Since $m_1 = m_2$

$\therefore$ The lines are parallel

(d) Find the equation of straight line parallel to 5x+6y-10=0 and passing through the Point (-3, 3) 5

Sol^n

$$5x+6y-10=0 \quad \& \ (x_1, y_1) = (-3, 3)$$

$$\Rightarrow 5x_1 + 6y_1 + K = 0$$

$$\Rightarrow 5(-3) + 6(3) + K = 0$$

$$\Rightarrow -15 + 18 + K = 0$$

$$\Rightarrow 3 + K = 0$$

$$\Rightarrow K = -3$$

$$\Rightarrow \boxed{5x + 6y - 3 = 0}$$

OR

Find the equation of the line cutting off equal intercepts and passing through the point (-2, 5)

Sol^n

$$\boxed{\frac{x}{a} + \frac{y}{b} = 1}$$

$$\frac{x}{a} + \frac{y}{a} = 1$$

$$\frac{x}{3} + \frac{y}{3} = 1$$

$$\Rightarrow \frac{x+y}{3} = 1$$

$$\frac{-2}{a} + \frac{5}{a} = 1$$

$$\frac{1}{a}(-2+5) = 1$$

$$\frac{3}{a} = 1$$

$$\Rightarrow \boxed{a = 3}$$

$$\Rightarrow \boxed{x + y = 3}$$

$$\Rightarrow \boxed{x + y - 3 = 0}$$

40/40

1 & 2

Course

$\Rightarrow$

3. (a) Convert 120^0 into radian and $\frac{3\pi}{2}$ into degree **4**

OR

Sol^n

$$120^0 = \cancel{120}^{2} \times \frac{\pi}{\cancel{180}_{3}} = \frac{2\pi}{3}$$

$$\frac{3\pi}{2} = \frac{3(\cancel{180}^{90})}{\cancel{2}} = 270^0$$

Prove that $\sin 2A = 2\sin A\cos A$

Sol^n

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$\Rightarrow$ put $B = A$

$$\sin(A+A) = \sin A\cos A + \cos A\sin A$$

$$\boxed{\sin 2A = 2\sin A\cos A}$$

Prove that

(b) Prove that $\tan(45^0 + A) = \dfrac{1 + \tan A}{1 - \tan A}$ 6

OR

Sol^n

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

put, $A = 45^\circ$, $B = A$

$$\tan(45^\circ + A) = \frac{\tan 45^\circ + \tan A}{1 - \tan 45^\circ \tan A}$$

$$\boxed{= \frac{1 + \tan A}{1 - \tan A}}$$

OR

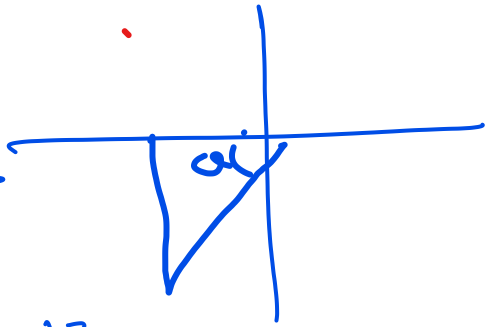
If $\tan A = \frac{5}{12} = \frac{O}{A}$ and $180^0 < A < 270^0$ then find the value of $\sin A$ *and* $\cos A$

Sol^n

$O = 5$, $A = 12$

$H = \sqrt{O^2 + A^2} = \sqrt{25 + 144}$

$H = \sqrt{169} = 13$



$$\boxed{\begin{aligned} \sin A &= \frac{O}{H} = -\frac{5}{13} \\ \cos A &= \frac{A}{H} = -\frac{12}{13} \end{aligned}}$$

(c) Simplify $\dfrac{\cos(360^0 - A)\tan(360^0 + A)}{\cot(270^0 - A)\sin(90^0 + A)}$ 5

OR

$$GE = \frac{\cos(360-A) \times \tan(360^{\circ}+A)}{\cot(270^{\circ}-A) \times \sin(90+A)}$$

$$= \left\{ \frac{\cancel{\cos} A \times \cancel{\tan A}}{\cancel{\tan} A \times \cos \cancel{A}} \right\}$$

$$= \underline{\underline{1}}$$

OR

Prove that $\frac{1-\cos 2A+\sin 2A}{1+\cos 2A+\operatorname{Sin} 2A}=\tan A.$

Sol^n

$$LHS = \frac{1-\cos 2A+\sin 2A}{1+\cos 2A+\sin 2A}$$

$$= \frac{2\sin^2 A + 2\sin A\cos A}{2\cos^2 A + 2\sin A\cos A}$$

$$= \frac{\cancel{2}\sin A\,\cancel{(\sin A+\cos A)}}{\cancel{2}\cos A\,\cancel{(\cos A+\sin A)}}$$

$$= \tan A = RHS$$

(d) Show that $\cos 100^0 + \cos 80^0 = 0$

OR

$$LHS = \cos 100 + \cos 80$$
$$= \cos 100 + \cos(180 - 100)$$
$$= \cos \cancel{100} - \cancel{\cos 100} = 0$$
$$= 0 = RHS$$

OR

Show that $\underline{\cos 20^0 \cos 40^0 \cos 80^0} = \frac{1}{8}$.

Sol^n

$$\text{LHS} = \underline{\cos 80 \cos 40} \cos 20$$

$$= \frac{1}{2}\left[\underline{\cos(120)} + \cos(40)\right] \cos 20$$

$$= \frac{1}{2}\left[-\frac{1}{2} + \cos 40\right] \cos 20$$

$$= +\frac{1}{2}\left[-\frac{1}{2}\cos 20 + \underline{\cos 40 \cos 20}\right]$$

$$= \frac{1}{2}\left[-\frac{1}{2}\cos 20 + \frac{1}{2}\left(\cos 60 + \cos 20\right)\right]$$

$$= \frac{1}{2}\left[\cancel{-\frac{1}{2}\cos 20} + \frac{1}{2}\underline{\cos 60} + \cancel{\frac{1}{2}\cos 20}\right]$$

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8} = \text{RHS}$$ //

500

$y_1 = y' = f'(x)$

4. (a) If $y = x^3 + 3\cos x + 4e^x + 2$ then find $\frac{dy}{dx}$.

Soln

$$\boxed{y' = 3x^2 + 3(-\sin x) + 4e^x + 0}$$

4 m

If $y=(x+1)(x-1)$ then find $\frac{dy}{dx}$.

Soln

$y = x^2 - 1^2$

$y = x^2 - 1$

$y' = 2x - 0$

$\boxed{y' = 2x}$

6

(b) Find the maximum and minimum value of a function $y = 2x^3 - 15x^2 + 36x + 6$

OR

$y = 2x^3 - 15x^2 + 36x + 6$

$y' = 2(3x^2) - 15(2x) + 36(1) + 0$

$y' = 6x^2 - 30x + 36$

$y'' = 6(2x) - 30(1)$

$y'' = 12x - 30$

$y' = 0$

$\Rightarrow \quad 6x^2 - 30x + 36 = 0$

$\boxed{x_1 = 3} \qquad \boxed{x_2 = 2}$

$\Rightarrow \quad x = 3$

$y'' = 12(3) - 30 = 36 - 30 = 6 > 0$

Minima

$y = 2(3)^3 - 15(3)^2 + 36(3) + 6$

$= 33$

$y'' = 12(2) - 30 = 24 - 30 = -6 < 0$

Maxima

$$y = 2(2)^3 - 15(2)^2 + 36(2) + 6$$

$$= 34$$

If $s = t^3 - 2t^2 + 6t + 8$ is the equation of motion of a particle in meters, find the acceleration at the end of 3 secs

Sol^n

$$s = t^3 - 2t^2 + 6t + 8$$

$$v = s' = 3t^2 - 4t + 6$$

$$v = 3(3)^2 - 4(3) + 6$$

$$= 27 - 12 + 6$$

$$\boxed{= 33 - 12 = 21}$$

$$a = s'' = 3(2t) - 4(1)$$

$$= 6t - 4$$

$$\boxed{a = 6(3) - 4 = 18 - 4 = 14}$$

If $y = a\cos mx + b\sin mx$ then prove that $\frac{d^2y}{dx^2} + m^2 y = 0.$

OR

Sol^n

$\Rightarrow \quad y = a\cos mx + b\sin mx$

$y' = -m(a\sin mx + b\cos mx)$

$y'' = -m^2 \underline{(a\cos mx + b\sin mx)}$

$\Rightarrow \quad y'' = -m^2 y$

$\Rightarrow \quad \boxed{y'' + m^2 y = 0}$

Find the derivative of a function $\frac{1+\sin x}{1-\sin x}$ w.r.t.x.

$$y = \frac{1+\sin x}{1-\sin x} \qquad \frac{u}{v}$$

$u = 1 + \sin x$

$u' = 0 + \cos x$

$v = 1 - \sin x$

$v' = 0 - \cos x$

$$y' = \frac{vu' - uv'}{v^2}$$

$$= \frac{(1-\sin x)\cos x - (1+\sin x)(-\cos x)}{(1-\sin x)^2}$$

Find the equation of tangent to the curve $y = 1 - x^3$ at the point (2, 3)

OR

Sol^n

$$y = 1 - x^3$$

$$m = y' = 0 - 3x^2$$

$$m = -3(2)^2$$

$$\boxed{m = -12}$$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = -12(x - 2)$$

$$y - 3 = -12x + 24$$

$$\Rightarrow \boxed{12x + y - 27 = 0}$$

OR

If $y = (1 + x^2)\tan^{-1} x$ then find $\frac{dy}{dx}$.

Sol^n

$y' = uv' + vu'$

$u = 1 + x^2$ $\qquad$ $v = \tan^{-1} x$

$u' = 2x$ $\qquad$ $v' = \frac{1}{1+x^2}$

$y' = \cancel{(1+x^2)} \times \frac{1}{\cancel{1+x^2}} + \tan^{-1} x (2x)$

$$\boxed{y' = 1 + 2x \tan^{-1} x}$$

5. (a) Evaluate $\int \tan^2 x\,dx$

Sol^n

$$I = \int \tan^2 x \, dx$$

$$= \int (\sec^2 x - 1)\, dx$$

$$= \tan x - x + c$$

OR

Integrate the function $\sin x + \frac{1}{x} + x^3 - 7$ w.r.t.x.

$$I = \int \left(\sin x + \frac{1}{x} + x^3 - 7 \right) dx$$

$$= -\cos x + \log x + \frac{x^4}{4} - 7x + c$$

(b) Find area bounded by the curve $y = x^2 + 2$, the x-axis and the ordinates at x=1 and x=2 (6)

OR

Sol^n

$$A = \int_a^b y\,dx$$

$$= \int_1^2 (x^2+2)\,dx$$

$$= \left[\frac{x^3}{3} + 2x\right]_1^2$$

$$= \left(\frac{(2)^3}{3} + 2(2)\right) - \left(\frac{1}{3} + 2\right)$$

$$= \left(\frac{8}{3} + 4\right) - \left(\frac{7}{3}\right)$$

$$= \frac{20}{3} - \frac{7}{3} = \frac{13}{3}$$ //

OR

Find the volume of the solid generated by revolving the line $y^2=2x+1$ about x-axis between the ordinates x=0 and x=2

Sol^n

$$V = \int_a^b \pi y^2 \, dx$$

$$= \pi \int_0^2 (2x+1) \, dx$$

$$= \pi \left[\frac{\cancel{2}x^2}{\cancel{2}} + x \right]_0^2$$

$$= \pi \left[\left((2)^2 + 2 \right) - (0+0) \right]$$

$$= 6\pi$$

(c) Evaluate the indefinite integral $\int (x \sin x)dx$ using integration by parts. 5

OR

$$I = \int x \cdot \sin x \, dx \qquad I\ L\ \underline{A}\ T\ E$$

$$\int uv \, dx = uv_1 - \int v_1 u' \, dx$$

$$= x(-\cos x) - \int (-\cos x)(1) \, dx$$

$$\boxed{I = -x\cos x + \underline{\sin x} + c}$$

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(d) Evaluate the indefinite integral $\int \frac{x+1}{x^2+2x+1} dx$ using integration by substitution method. 5

OR

Sol^n

$$I = \int \frac{x+1}{x^2+2x+1} dx$$

$$= \frac{1}{2} \int \frac{2x+2}{x^2+2x+1} dx$$

$$\int \frac{f'(x)}{f(x)} dx = \log f(x) + c$$

$$\boxed{= \frac{1}{2} \log(x^2+2x+1) + c}$$

Evaluate $\int \sin^3 x \, dx$

Sol^n $\sin 3x = 3\sin x - 4\sin^3 x$

$$4\sin^3 x = 3\sin x - \sin 3x$$

$$\sin^3 x = \frac{1}{4}\left[3\sin x - \sin 3x\right]$$

$$\int \sin^3 x \, dx = \frac{1}{4}\int (3\sin x - \sin 3x) \, dx$$

$$= \frac{1}{4}\left[-3\cos x + \frac{\cos 3x}{3}\right] + c$$

$$\boxed{I = \frac{1}{4}\left[-3\cos x + \frac{\cos 3x}{3}\right] + c}$$