1344 **Code : 20SC51T**

Register Number | | | | | | | | | | |

V Semester Diploma Examination, June/July-2023

APPLIED MATHEMATICS

Time : 3 Hours] **[Max. Marks : 100**

Instructions : (i) Solve any **five** full questions, choosing **one** full question from each section.

(ii) Each full question carries **20** marks.

SECTION – I

1. (a) Find the angle between the given pairs of curves $r = \frac{a}{1+\cos\theta}$ and $r = \frac{b}{1-\cos\theta}$ **10**

   (b) Find the Pedal equation of the curve $r^m = a^m \cos m\theta$ **10**

2. (a) Show that the pairs of curves $r = a(1 + \sin\theta)$ and $r = a(1 - \sin\theta)$ intersect orthogonally. **10**

   (b) Find the radius of curvature of the curve $x^3 + y^3 = 3axy$ at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ on it. **10**

SECTION – II

3. (a) Find the Jacobian $J = \frac{\partial(u,v,w)}{\partial(x,y,z)}$ where $u = x^2 + y^2 + z^2$; $v = xy + yz + xz$; $w = x + y + z$ **8**

   (b) Solve : $(5x^4 + 3x^2y^2 - 2xy^3)\,dx + (2x^3y - 3x^2y^2 - 5y^5)\,dy = 0$ **6**

   (c) Using the methods of variation of parameters, solve : $\frac{d^2y}{dx^2} + y = \tan x$ **6**

4. (a) Using Maclaurin's series prove that :

$$\sqrt{1+\sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} + \ldots\ldots$$ **8**

(b) Using Clairaut's equation show that $xy^3 - yp^2 + 1 = 0$ **6**

(c) Solve : $\frac{dy}{dx} + \frac{y}{x} = x^3$ **6**

SECTION – III

5. (a) Solve the following system of equation by Gauss – Elimination method. **10**

$x + y + z = 9$;

$2x + y + z = 0$;

$2x + 5y + 7z = 52$;

(b) Prove that

$$\beta(m, n) = \frac{\lceil m \rceil \lceil n}{\lceil m+n}$$ **10**

6. (a) Solve by Gauss – Jordan method :

$x + y + z = 10$;

$2x - y + 3z = 19$;

$x + 2y + 3z = 22$ **10**

(b) Evaluate : $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x \, dz\,dx\,dy$ **10**

SECTION – IV

7. (a) If $\vec{F} = \nabla(xy^3z^2)$ find curl $(\vec{F})$ at $(1, -1, 1)$ **6**

(b) Form the PDE by eliminating arbitrary constants a and b from

$(x - a)^2 + (y - b)^2 + z^2 = 4$ **7**

(c) Solve : $xp + yq = 3z$ **7**

8. (a) Show that :

$\vec{F} = (y^2 - z^2 + 3yz - 2x)i + (3xz + 2xy)j + (3xy - 2xz + 2z)k$ is solenoid. **7**

(b) Using Green's theorem, Find the area between parabolas $y^2 = 4x$ and $x^2 = 4y$ **6**

(c) Derive one Dimensional wave equation. **7**

SECTION – V

9. (a) Using Lagrange's Interpolation formula fit a polynomial for following data to find y(3). **10**

x	0	1	2	5
y	2	3	12	147

(b) Use Taylor's series method to find y(0.1) and y(0.2) upto fourth degree, given that $\frac{dy}{dx} = x^2y - 1$ and $y(0) = 1$ **10**

10. (a) Evaluate $\int_0^{0.6} e^{-x^2}\,dx$ by using Simpson's $1/3^{rd}$ rule by taking six parts. **10**

(b) Compute y(0.2) using Runge-Kutta method of fourth order by taking h = 0.2 given that $\frac{dy}{dx} = 3x + \frac{y}{2}$ and $x_0 = 0$, $y_0 = 1$ **10**
