V Semester Diploma Make up Examination,

September – 2023

COURSE NAME: APPLIED MATHEMATICS

COURSE CODE: 20SC51T

SCHEME & MODEL ANSWERS OF VALUATION

	SECTION-I	
1 (a)	**Find the angle between the radius vector and the tangent for the curve $r^2 = a^2 \cos 2\theta$.**	(10)
Solution	Given , $r^2 = a^2 \cos 2\theta$	
	WKT,Angle between the radius vector and the tangent for the curve is $\tan\phi = r.\frac{d\theta}{dr}$	1
	Take log on both sides $\log r^2 = \log(a^2 \cos 2\theta)$ $\log r^2 = \log a^2 + \log \cos 2\theta$ $2\log r = 2\log a + \log \cos 2\theta$	2
	Differentiating wrt θ $\frac{d(2\log r)}{d\theta} = \frac{d(2\log a)}{d\theta} + \frac{d(\log\cos 2\theta)}{d\theta}$ $2.\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{1}{\cos 2\theta}.-\sin 2\theta.2$ $\frac{1}{r}\frac{dr}{d\theta} = \frac{-\sin 2\theta}{\cos 2\theta}$ $\frac{dr}{d\theta} = - r\frac{-\sin 2\theta}{\cos 2\theta}$ $\frac{d\theta}{dr} = -\frac{1}{r}.\frac{\cos 2\theta}{\sin 2\theta}$ $\frac{d\theta}{dr} = -\frac{1}{r}\cot 2\theta$	4
	Now, $\tan\phi = r.\frac{d\theta}{dr}$ $\tan\varphi = r.\left(-\frac{1}{r}\cot 2\theta\right)$	1
	$\tan\phi = -\cot 2\theta$	1
	$\phi = \frac{\pi}{2} + 2\theta$	1

1 (b) **Find the Pedal equation of the curve $\frac{2a}{r} = 1 + \cos\theta$** **(10)**

Solution:

Given

$1 + \cos\theta$……..(1)

WKT, Pedal Equation of the curve is ,

$\boldsymbol{p = r\, sin\emptyset}$

1

$$\frac{2a}{r} = 2cos^2\frac{\theta}{2}$$

1

$$\boldsymbol{cos\frac{\theta}{2} = \sqrt{\frac{a}{r}}}$$ 1

Take log on both sides for (1)

$log2a - \log r = \log(1 + cos\theta)$ 1

Differentiating wrt θ

$$-\frac{1}{r}\frac{dr}{d\theta} = \frac{1}{1+cos\theta} \cdot -sin\theta$$

$$\frac{dr}{d\theta} = r.\frac{sin\theta}{1 + cos\theta}$$

$$\frac{d\theta}{dr} = \frac{1}{r} \cdot \frac{1 + cos\theta}{sin\theta}$$ 2

$$tan\emptyset = r.\frac{1}{r} \cdot \frac{1 + cos\theta}{sin\theta}$$

1

$$= \frac{1+cos\theta}{sin\theta} = \frac{2cos^2\frac{\theta}{2}}{2sin\frac{\theta}{2}cos\frac{\theta}{2}}$$

$$tan\varphi = cot\frac{\theta}{2}$$

$$\emptyset = \frac{\pi}{2} - \frac{\theta}{2}$$

1

Now,

$p = r\, sin\varphi$

$$= r\sin\left(\frac{\pi}{2} - \frac{\theta}{2}\right)$$

$$= rcos\frac{\theta}{2} = r\sqrt{\frac{a}{r}}$$

$p = \sqrt{ar}$ 1

$\boldsymbol{p^2 = ar}$ 1

2 (a)	**Show that the pair of curves r = ae^{θ} , re^{θ} = b intersect each other orthogonally.**	**(10)**
Solution:	$r = ae^{\theta}$; $re^{\theta} = b$	
	Take log on both sides ; Take log on both sides	
	$\log r = \log a + \log e^{\theta}$; $\log r + \log e^{\theta} = \log b$	3
	$\log r = \log a + \theta$; $\log r + \theta = \log b$	
	Differentiating wrt θ ; Differentiating wrt θ	
	$\frac{1}{r}\frac{dr}{d\theta} = 1$; $\frac{1}{r}\frac{dr}{d\theta} + 1 = 0$	
	$r\frac{d\theta}{dr} = 1$; $r\frac{d\theta}{dr} = -1$	4
	$tan\emptyset = r.\frac{d\theta}{dr}$; $tan\emptyset = r.\frac{d\theta}{dr}$	
	$tan\emptyset_1 = 1$; $tan\emptyset_2 = -1$	
	$\emptyset_1 = \frac{\pi}{4}$; $\emptyset_2 = -\frac{\pi}{4}$	
	$Now,$	
	$\alpha = \lvert\emptyset_2 - \emptyset_1\rvert$	
	$= \left\lvert -\frac{\pi}{4} - \frac{\pi}{4} \right\rvert$	1
	$\boldsymbol{\alpha = \frac{\pi}{2}}$	1
	Thus,	
	Pair of curves intersect each other orthogonally.	1

2 (b)	**Find the radius of curvature of the curve x = acost , y = asint.**	**(10)**
Solution:	Given x = acost , y = asint WKT, Radius of curvature of the curve is, $\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$	**2**
	Differentiating wrt t,we get $\frac{dx}{dt} = -asint$, $\frac{dy}{dt} = acost$	**1**
	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ $= \frac{acost}{-asint}$	**1**
	$\frac{dy}{dx} = -cott$	**1**
	Differentiating again wrt x,we get $\frac{d^2y}{dx^2} = -(-cosec^2t)\frac{dt}{dx}$ $=cosec^2t.-\frac{1}{asint}$	**2**
	$\frac{d^2y}{dx^2} = \frac{-cosec^3t}{a}$ Now, $\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$ $= \frac{[1+(-cott)^2]^{\frac{3}{2}}}{-\frac{cosec^3t}{a}}$	**2**
	$= \frac{[1+cot^2t]^{\frac{3}{2}}}{-\frac{cosec^3t}{a}}$ $= (cosec^2t)^{\frac{3}{2}} . \frac{-a}{cosec^3t}$ $\rho = -a$ i.e $\rho = a$	**1**
	SECTION-II	

3 (a) **Solution:**	**Obtain Maclaurin's series expansion of log $(1+x)$ upto x^4** $f(x) = \log_e(1+x)$ *Maclaurin's Series expansion is given by* $f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \ldots\ldots$ $f(x) = \log_e(1+x)$ $f(0) = \log_e(1+0) = 0$ $f'(x) = \frac{1}{(1+x)}$ $f'(0) = \frac{1}{(1+0)} = 1$ $f''(x) = \frac{-1}{(1+x)^2}$ $f''(0) = \frac{-1}{(1+0)^2} = -1$ $f'''(x) = \frac{2}{(1+x)^3}$ $f'''(0) = \frac{2}{(1+0)^3} = 2$ $f^{IV}(x) = \frac{-6}{(1+x)^4}$ $f^{IV}(0) = \frac{-6}{(1+0)^4} = -6$ *Maclaurin's series expansion is* $\mathbf{\log_e(1+x) = x - \frac{x^2}{2!} + 2\frac{x^3}{3!} - 6\frac{x^4}{4!} + \cdots \ldots\ldots\ldots}$	**(8)** 2 1 1 1 1 1 1
3 (b) **Solution:**	**Solve $(4xy + 3y^2 - x)dx + x(x+2y)dy = 0$** $(4xy + 3y^2 - x)dx + x(x+2y)dy = 0$ $M = 4xy + 3y^2 - x$, $N = x^2 + 2xy$ $\frac{\partial M}{\partial y} = 4x + 6y$, $\frac{\partial N}{\partial x} = 2x + 2y$ $\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 4x + 6y - 2x - 2y$ $= 2x + 4y$ $= 2(x+2y)$ ---→ Close to N Now $\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{2(x+2y)}{x(x+2y)} = \frac{2}{x} = f(x)$ Integrating Factor = $e^{\int f(x)dx} = e^{\int \frac{2}{x}dx} = e^{2logx} = x^2$ Multiply given equation by x^2 $M = 4x^3y + 3x^2y^2 - x^3$, $N = x^4 + 2x^3y$ $\frac{\partial M}{\partial y} = 4x^3 + 6x^2y$, $\frac{\partial N}{\partial x} = 4x^3 + 6x^2y$ Soln is ,	**(6)** 1 1 1 1

	$\int Mdx + \int Ndy = C$ $\int_{y-const} (4x^3y + 3x^2y^2 - x^3)\, dx + \int 0.dy = C$ $\boldsymbol{x^4y + x^3y^2 + \frac{x^4}{4} = C}$	**2**
3 (c) **Solution:**	**Solve $(D^2 + 5D + 6) = e^x$** Given, $(D^2 + 5D + 6) = e^x$ The Auxiliary Equation is, $m^2 + 5m + 6 = 0$ $m^2 + 3m + 2m + 6$ $m = -3, -2$ *are the roots which are real and distinct* Complimentary Function, $C.F = c_1e^{-2x} + c_2e^{-3x}$ Now, Particular Integral, $P.I = \frac{1}{f(D)}e^x$ $= \frac{1}{D^2+5D+6}\, e^x$ Put D=1 $= \frac{1}{1^2+5.1+6}\, e^x$ $P.I = \frac{e^x}{12}$ The Solution is, $\boldsymbol{y = C.F + P.I}$ $\boldsymbol{y = c_1e^{-2x} + c_2e^{-3x} + \frac{e^x}{12}}$	**(6)** **1** **1** **1** **1** **1** **1**

4 (a) **Expand $\sin(e^x - 1)$ using Maclaurin's series expansion upto the term containing x^4** **(8)**

Solution:

Maclaurin's Series expansion is

$$f(x) = f(0) + \frac{x}{1!}f'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \frac{x^4}{4!}f^{IV}(0) + \cdots. \quad \mathbf{1}$$

$f(x) = \sin(e^x - 1)$ $\qquad f(0) = 0$ **1**

$f'(x) = \cos(e^x - 1)\, e^x$ $\qquad f'(0) = 1$ **1**

$f''(x) = \cos(e^x - 1)\, e^x + e^x[-\sin(e^x - 1)]e^x$ $\qquad f''(0) = 1$ **1**

$f''(x) = e^x[\cos(e^x - 1) - e^x \sin(e^x - 1])$

$f''(x) = f'(x) - e^{2x} f(x)$

$f''(x) = f'(x) - e^{2x} y$

$f'''(x) = f''(x) - e^{2x} f'(x) - f(x)e^{2x}.2$ **1**

$f'''(0) = 1 - e^{2(0)} \times 1 - 2(0).1$

$f'''(0) = 0$

$f^{IV}(x) = f'''(x) - e^{2x} f''(x) - f'(x)e^{2x}.2 - 2ye^{2x}.2 - 2e^{2x} f'(x)$

$f^{IV}(0) = 0 - 1 - 2 - 4(0).1 - (2).1$ **1**

$f^{IV}(0) = -5$

∴ Maclaurin's Series expansion is **1**

$$\boldsymbol{f(x) = \sin(e^x - 1) = x + \frac{x^2}{2!} - 5\frac{x^4}{4!} + \cdots} \quad \mathbf{1}$$

4 (b) **Solve $\tan y \frac{dy}{dx} + \tan x = \cos y \cos^2 x$** **(6)**

Solution:

$\tan y \frac{dy}{dx} + \tan x = \cos y \cos^2 x$

Dividing both sides by cosy

$sec y . tan y \frac{dy}{dx} + tan x sec y = \cos^2 x$ **1**

$put\ sec y = t$

$sec y tan y \frac{dy}{dx} = \frac{dt}{dx}$ **1**

$\frac{dt}{dx} + tan x . t = \cos^2 x$

$P = tan x \ , Q = \cos^2 x$

I.F $= e^{\int P dx} = e^{\int tan x dx} = e^{log sec x} = sec x$ **2**

$t . sec x = \int \cos^2 x . sec x dx + C$ **1**

	$\boldsymbol{secx.secy = sinx + C}$	**1**
4 (c) **Solution:**	**Solve $\frac{d^2y}{dx^2}$ +y =secx tanx using variation of parameters** Given, $\frac{d^2y}{dx^2}$ +y =secx tanx We have, $(D^2+1)y = secx\ tanx$ A.E is, $m^2+1=0$ $m=\pm i$ $C.F = c_1 cosx + c_2 sinx$ $y_1 = cosx\ ,\ y_2 = sinx$ $P.I = -y_1 \int \frac{y_2 X}{W} dx + y_2 \int \frac{y_1 X}{W} dx$ where, $X = secx.tanx$ & $W = \begin{vmatrix} cosx & sinx \\ -sinx & cosx \end{vmatrix} = cos^2x + sin^2x = 1$ $P.I = -cosx \int \frac{sinx.secxtanx}{1} dx + sinx \int \frac{cosx.secxtanx}{1} dx$ $= -cosx \int tan^2x\, dx + sinx \int tanx\ dx$ $= -cosx \int (sec^2x - 1) dx + sinx \int tanx\ dx$ $= -cosx(tanx - x) + sinx.logsecx$ The solution is, $\boldsymbol{y = C.F + P.I}$ $\boldsymbol{y = c_1 cosx + c_2 sinx - cosx(tanx - x) + sinx.logsecx}$	**(6)** **1** **1** **1** **1** **1** **1**

SECTION-III

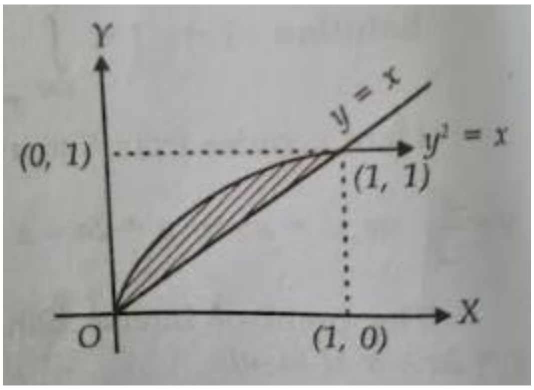
5 (a) **Solve the following system of equation by Gauss_Elimination method :** **(10)**

Solution: ***Typing error---Give grace marks those who have attempted it.**

5 (b) **Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy\, dydx$ by changing order of integration.** **(10)**

Solution:

For the required region x varies from 0 to 1 and y varies from y=x to y=$\sqrt{x}$ i.e $y^2 = x$ which is a parabola. **1**



1

$y = x$, $y = \sqrt{x} \Rightarrow y^2 = x$

$x^2 = x$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0 , x = 1$

$\therefore y = 0 , y = 1$

The point of intersection of y=x with the parabola $y^2 = x$ is (1,1) **2**

$x = y^2,\ x = y\ ,\ y = 0\ ,\ y = 1$ **1**

$$I = \int_{y=0}^{y=1} \int_{x=y^2}^{x=y} xydxdy$$

$$I = \int_{y=0}^{y=1} \left[y\frac{x^2}{2}\right]_{y^2}^{y} dy$$

$$I = \int_{y=0}^{y=1} \left[y.\frac{y^2}{2} - y.\frac{(y^2)^2}{2}\right] dy$$ **1**

$$= \frac{1}{2}\int_{y=0}^{y=1} (y^3 - y^5)\, dy$$ **1**

$$= \frac{1}{2}\left[\frac{y^4}{4} - \frac{y^6}{6}\right]_0^1$$ **1**

$$= \frac{1}{2}\left[\frac{1}{4} - \frac{1}{6}\right]$$

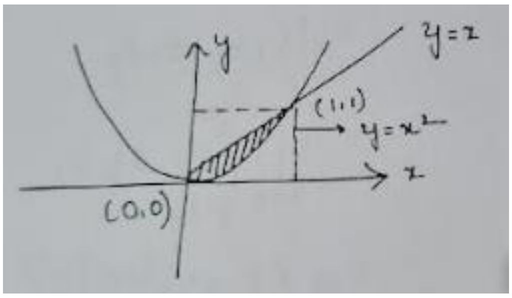
$$= \mathbf{\frac{1}{24}}$$ **2**

6 (a)	**Solve by Gauss-Jordan Method** **x +2y + z = 8** **2x + 3y + 4z = 20** **4x + 3y + 2z = 16**	**(10)**
Solution:	Given, x +2y + z = 8 2x + 3y + 4z = 20 4x + 3y + 2z = 16	
	The augmented matrix is, $[A:B]=\left(\begin{array}{ccc|c}1&2&1&8\\2&3&4&20\\4&3&2&16\end{array}\right)$	**2**
	$R_2=R_2-2R_1$, $R_3=R_3-4R_1$ $[A:B]=\left(\begin{array}{ccc|c}1&2&1&8\\0&-1&2&4\\0&-5&-2&-16\end{array}\right)$	**2**
	$R_3=R_3-5R_2$ $[A:B]=\left(\begin{array}{ccc|c}1&2&1&8\\0&-1&2&4\\0&0&-12&-36\end{array}\right)$	**2**
	$R_1=R_1+2R_2$ $[A:B]=\left(\begin{array}{ccc|c}1&0&5&16\\0&-1&2&4\\0&0&-12&-36\end{array}\right)$	**1**
	$R_3=R_3/-12$, $R_2=R_2\text{x}-1$ $[A:B]=\left(\begin{array}{ccc|c}1&0&5&16\\0&1&-2&-4\\0&0&1&3\end{array}\right)$	**1**
	$R_1=R_1-5R_3$, $R_2=R_2+2R_3$ $[A:B]=\left(\begin{array}{ccc|c}1&0&0&1\\0&1&0&2\\0&0&1&3\end{array}\right)$ is an identity matrix	**1**
	$\boldsymbol{Hence\ x=1\ , y=2\ \ , z=3}$	**1**

6 (b)	Evaluate $\int_{x=0}^{x=1}\int_{y=0}^{y=\sqrt{1-x^2}}\int_{z=0}^{z=\sqrt{1-x^2-y^2}} xyz\,dz\,dy\,dx$	(10)
Solution:	$$I=\int_{x=0}^{x=1}\int_{y=0}^{y=\sqrt{1-x^2}}\int_{z=0}^{z=\sqrt{1-x^2-y^2}} xyz\,dz\,dy\,dx$$	
	$=\int_{x=0}^{x=1}\int_{y=0}^{y=\sqrt{1-x^2}} xy\left[\frac{z^2}{2}\right]_0^{\sqrt{(1-x^2-y^2)}} dydx$	1
	$=\int_{x=0}^{x=1}\int_{y=0}^{y=\sqrt{1-x^2}} xy\frac{(1-x^2-y^2)}{2}dydx$	1
	$=\int_{x=0}^{x=1}\int_{y=0}^{y=\sqrt{1-x^2}}(xy-x^3y-xy^3)dydx$	1 1
	$=\frac{1}{2}\int_{x=0}^{x=1}\left[x\frac{y^2}{2}-x^3.\frac{y^2}{2}-x.\frac{y^4}{4}\right]_0^{\sqrt{1-x^2}} dx$	
	$=\frac{1}{2}\int_{x=0}^{x=1}\left[x\frac{1-x^2}{2}-x^3\frac{1-x^2}{2}-\frac{x}{4}(1-x^2)^2\right]dx$	1
	$=\frac{1}{2}\int_{x=0}^{x=1}\left[\frac{x-x^3}{2}-\frac{x^3+x^5}{2}-\frac{x}{4}(1+x^4-2x^2)\right]dx$	
	$=\frac{1}{4}\int_{x=0}^{x=1}\left[x-x^3-x^3+x^5-\frac{x+x^5-2x^3}{2}\right]dx$	2
	$=\frac{1}{8}\int_{x=0}^{x=1}[2x-4x^3+2x^5-x-x^5+2x^3]\,dx$	1
	$=\frac{1}{8}\int_{x=0}^{x=1}[x-2x^3+x^5]\,dx$	1
	$=\frac{1}{8}\left[\frac{x^2}{2}-\frac{2x^4}{4}+\frac{x^6}{6}\right]_0^1$	
	$=\frac{1}{8}\left[\frac{1}{2}-\frac{1}{2}+\frac{1}{6}\right]$	
	$=\frac{1}{48}$	1
	SECTION-IV	
7 (a)	If $\vec{F}=(x+y+1)i+j-(x+y)k$ then show that $\vec{F}.curl\vec{F}=0$	(6)
Solution:	$$\vec{F}=(x+y+1)i+j-(x+y)k$$ $=f_1i+f_2j+f_3k$	1

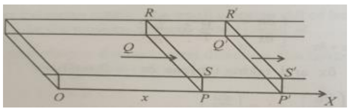
	$curl\vec{F} = \begin{vmatrix} i & j & k \\ \frac{\delta}{\delta x} & \frac{\delta}{\delta y} & \frac{\delta}{\delta z} \\ x+y+1 & 1 & -x-y \end{vmatrix}$	**2**
	$= i[-1-0] - j[-1-0] + k[0-1]$	**1**
	$= -i + j - k$	**1**
	$\vec{F}.curl\vec{F} = -x - y - 1 + 1 + x + y$	
	=0	**1**
7 (b)	**Form the PDE by elimininating arbitrary constants a and b from** $\mathbf{z = (x-a)^2 + (y-b)^2}$	**(6)**
Solution:	Given, $\mathbf{z = (x-a)^2 + (y-b)^2}$**………………..(1)** Differentiating partially wrt x, $\frac{\partial z}{\partial x} = 2(x-a)(1-0) + 0$ $p = 2(x-a)$ $\frac{p}{2} = (x-a)$	**2**
	Differentiating partially wrt y, $\frac{\partial z}{\partial y} = 0 + 2(y-b)(1-0)$ $\frac{\partial z}{\partial y} = 2(y-b)$ $q = 2(y-b)$ $\frac{q}{2} = (y-b)$	**2**
	Substituing in (1)we get, $z = \left(\frac{p}{2}\right)^2 + \left(\frac{q}{2}\right)^2$	**1**
	$\mathbf{4z = p^2 + q^2}$ **is the required solution**	**1**

7 (c)	**Solve p + q = $\frac{z}{a}$ by Lagrange's method.**	**(8)**
Solution:	Given, p + q = $\frac{z}{a}$ is of the form $P_p + Q_q = R$	**1**
	The auxiliary equation is , $\frac{dx}{p} = \frac{dy}{q} = \frac{dx}{R}$	**2**
	$\frac{dx}{1} = \frac{dy}{1} = \frac{dx}{z/a} \dots\dots\dots\dots\dots\dots\dots\dots (1)$ Consider $\frac{dx}{1} = \frac{dy}{1}$ The solution on integrating is , $x = y + c$ $x - y = c_1$	**1** **1**
	Now consider, $\frac{dy}{1} = \frac{dx}{\frac{z}{a}}$	**1**
	The solution on integrating is, $y = alogz$ $y - alogz = c_2$	**1**
	The complete solution is, $\emptyset(u, v) = 0$ $\boldsymbol{\emptyset(x - y, y - alogz) = 0}$	**1**
8 (a)	**Find the directional derivatives of $\boldsymbol{\phi = xy^2 + yz^3}$ at $\boldsymbol{(2, -1, 1)}$ along $\boldsymbol{i + 2j + 2k}$**	**(7)**
Solution:	: $\phi = xy^2 + yz^3$ Grad $\phi = \nabla \phi = \frac{\delta\phi}{\delta x} i + \frac{\delta\phi}{\delta y} j + \frac{\delta\phi}{\delta z} k$ $= y^2 i + (2xy + z^3) j + 3yz^2 k$ $\nabla \phi_{(2,-1,1)} = (-1)^2 i + [2(2)(-1) + (1)^3] j + 3(-1)(1)^2 k$ $= i - 3j - 3k$	**1** **1**

	Unit vector normal along the direction $i+2j+2k$ $\hat{n}=\frac{\vec{d}}{\lvert\vec{d}\rvert}=\frac{(i+2j+2k)}{\sqrt{1+4+4}}=\frac{i+2j+2k}{3}$ Required directional derivative $=\nabla\phi.\hat{n}$ $=(i-3j-3k).\frac{(i+2j+2k)}{3}$ $=\frac{1}{3}(1-6-6)$ $=-\frac{\mathbf{11}}{\mathbf{3}}$	1 2 2
8 (b) **Solution:**	**Evaluate** $\oint\int_C (xy+y^2)dx+x^2dy$ **.where is** C **is the closed curve of the region bounded by** $y=x$ **and** $y=x^2$ **using Green's Theorem.** $\oint_c Mdx+\oint_c Ndy=\oint(xy+y^2)dx+x^2dy$ $M=xy+y^2$ $N=x^2$ $\frac{\partial M}{\partial y}=x+2y$ $\frac{\partial N}{\partial x}=2x$ $\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}=2x-x-2y$ $\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}=x-2y$  The curve C bounded by $y=x$ & $y=x^2$ ∴ By the Green's Theorem W.K.T $\oint_c Mdx+\oint_c Ndy=\iint_R\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)dxdy$ $\Rightarrow I=\int_{x=0}^{1}\int_{y=x^2}^{x}(x-2y)dydx$ $=\int_{x=0}^{1}[xy-y^2]_{x^2}^{x}dx$ $=\int_{x=0}^{1}[(x^2-x^2)-(x^3-x^4)]dx$ $=\int_0^1(x^4-x^3)dx$ $=\left[\frac{x^5}{5}-\frac{x^4}{4}\right]_0^1$ $=\frac{1}{5}-\frac{1}{4}$ $=\frac{\mathbf{-1}}{\mathbf{20}}$	(6) 1 1 1 1 1 1

8 (c) **Derive one dimensional heat equation.** **(7)**

Solution:



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Consider a homogeneous bar of constantcross-sectional area A.

Let ρ be the density, s be the specific heat and K be the thermal conductivity of the material. Let the sides be insulated so that the stream lines of heat flow are parallel andperpendicular to the area A.

Let one end of the bar be taken as the origin O and the direction of the heat flow be the positive x-axis

Let $u = u(x, t$ be the temperature of the slab at a distance x from theorigin.

Consider an element of bar between the planes $PQRS$ and $P'Q'R'S'$ at adistancex& $x+\delta x$ from the end O. Let δu be the change in temperature in a slab of thickness x of the bar The mass of the element = $A\rho\delta x$

The quantity of heat stored in this slab element = $A\rho s\delta x\delta u$

Hence the rate of increase of heat in this slab element is

$$R = (A\rho s\delta x)\frac{\partial u}{\partial t} \text{ --------} > (1)$$

If R_I is the rate of inflow of heat and R_O is the rate of outflow of heat, we have

$$R_I = -KA\left[\frac{\partial u}{\partial x}\right]_x \text{ and}$$

2

$$R_O = -KA\left[\frac{\partial u}{\partial x}\right]_{x+\delta x} \text{ ---------} > (2)$$

1

Where the negative sign is due to empirical law (1)

Hence we have from (1) & (2)

$$R = R_I - R_O$$

$$i.e, A\rho s\delta x\frac{\partial u}{\partial t} = KA\left[\frac{\partial u}{\partial x}\right]_{x+\delta x} - KA\left[\frac{\partial u}{\partial x}\right]_x$$

$$\frac{\partial u}{\partial t} = \frac{K}{\delta s} = \left\{\frac{\left[\frac{\partial u}{\partial x}\right]_{x+\partial x} - \left[\frac{\partial u}{\partial x}\right]_x}{\partial x}\right\} \text{ -------------} > 3$$

1

Taking limit as $\delta x \to 0, RHS\ is\ equal\ to$

1

$$\frac{K}{\rho s}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) = \frac{K}{\partial s}\frac{\partial^2 u}{\partial x^2}$$

Further denoting $c^2 = \frac{K}{\rho s}$ which is called the diffusity of the substance, (3) becomes

	$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ $u_t = c^2 u_{xx}$ **is the one dimensional heat equation**	1
	SECTION-V	
9 (a)	**Compute the fourth root of 12 correct to 3 decimal places using Regula Falsi method**	**(10)**
Solution:	Let x=$\sqrt[4]{12}$ $x = 12^{\frac{1}{4}}$ *Taking the power 4 on both sides* $x^4 = 12$ $x^4 - 12 = 0$ The function is, $f(x) = x^4 - 12$ Now f(0)=-12 (-ve) f(1)=1-12 =- 11 (-ve) f(2)=16-12 = 4 (+ve) The root lies between 1 & 2 f(1.5)=-ve,f(1.7)=-ve ,f(1.8)=+ve The root lies between 1.7 & 1.8 Now, $x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$ $= \frac{1.7(10.4976) - 1.8(-3.6479)}{10.4976 - (-3.6479)}$ x_2 =1.861	1 1 1 2 1 1 2 1

9 (b) Using Lagrange's Interpolation formula,fit a polynomial for following Data find y at x = 4 **(10)**

x	0	1	2	5
y	2	3	12	147

Solution:

Given

X	**0**	**1**	**2**	**5**
y	2	3	12	147

Lagrange's Interpolation Formula for above table is **3**

$$y = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1$$

$$+\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$

$$y = \frac{(x-1)(x-2)(x-5)}{(0-1)(0-2)(0-5)}(2) + \frac{(x-0)(x-2)(x-5)}{(1-0)(1-2)(1-5)}(3)$$ **3**

$$+\frac{(x-0)(x-1)(x-5)}{(2-0)(2-1)(2-5)}(12) + \frac{(x-0)(x-1)(x-2)}{(5-0)(5-1)(5-2)}(147)$$ **3**

$$y = -\frac{1}{5}[x^3-8x^2+17x-10] + \frac{3}{4}[x^3-7x^2+10x] - 2[x^3-6x^2+5x] + \frac{147}{60}[x^3-3x^2+2x]$$

$$y(x) = x^3 + x^2 - x + 3$$

$$y(4) = 79$$ **1**

10 (a) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's $\frac{1}{3}$ rd rule taking 4 equal strips and hence deduce an appropriate value of π. **(10)**

Solution:

Given,a=0 , b=1 , n=4

$$h = \frac{b-a}{n} = \frac{1-0}{4}$$ **1**

Also given, $y = \int_0^1 \frac{dx}{1+x^2}$

x	0	1/4	1/2	3/4	1
y	1	16/17	4/5	16/25	1/2

2

WKT, Simpson's one-third rule:

$$\int_a^b y\,dx = \frac{h}{3}\,[(y_0+y_0) + 4(y_1+y_3) + 2(y_2+y_4)]$$ **2**

$$= \frac{1/4}{3}[(1+1/2) + 4(16/17+16/25) + 2(4/5)]$$ **1**

$$= 0.7854$$ **1**

To deduce the value of π

1

	$\int_0^1 \frac{dx}{1+x^2} = tan^{-1}x \ \ between(0,1)$ $= tan^{-1}1 - tan^{-1}0$ $= \frac{\pi}{4} - 0$ $0.7854 = \frac{\pi}{4}$ $\pi = 3.142$	1 1
10 (b) **Solution:**	Apply Runge Kutta fourth order method ,to find an appropriate value of y when x=0.2 given that $\frac{dy}{dx} = x + y$ and y = 1 when x= 0 Here $x_0 = 0, y_0 = 1, h = 0.2, f(x_0, y_0) = 1$ $k_1 = hf(x_0, y_0)$ $k_2 = hf(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2}k_1) = 0.2 \times f(0.1,1.1)$; $k_2 = 0.2400$ $k_3 = hf(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2}k_2) = 0.2 \times f(0.1,1.12)$ $k_3 = 0.2440$ $k_4 = hf(x_0 + h, y_0 + k_3) = 0.2 \times f(0.2,1.244)$; $k_4 = 0.2888$ $k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$ $k = \frac{1}{6}(0.2000 + 0.4800 + 0.4880 + 0.2888)$ $k = \frac{1}{6}(1.4568) = 0.2468$ Hence the required approximate value of y is $y + k = 1.2428$	**(10)** 2 1 1 1 2 2 1
	"Certified that the model answers prepared by us for code **20SC51T** and scheme of valuation are correct to my knowledge". **AWARD FULL MARKS FOR ALTERNATE METHODS OF SOLUTIONS.**	