SCHEME OF VALUATION.

I & II SEMESTER DIPLOMA EXAMINATION.

April/May-2024

Sub: Engineering Mathematics Code: 20SC01T.

Qn No.	Matter	Marks
1. (a)	Finding A	1
	Finding $A+A^T$	2
	Final answer	1.
	OR	
	Writing A+B in the order	1.
	Calculation & Simplification	2.
	Final answer	1
1. (b)	Formula	1
	Finding $\lvert A \rvert$	2
	Finding adj A	2
	Final answer	1
	OR	
	Finding $[A-\lambda I]$	2
	Finding characteristic eqn.	2
	Finding roots	2
1 (c)	Finding Δ	1
	Finding $\Delta 1$	1
	Finding $\Delta 2$	1
	Finding x & y.	2
	OR	
	Performing determinant	2
	Calculation / Simplification	2
	Final answer	1

Q. No.	Matter.	Marks
1. (d)	Performing AB	3
	Finding $(AB)^T$	2
	OR	
	Finding adj A	1
	Finding A(adj A)	1
	Finding $\lvert A \rvert$	1
	Finding $\lvert A \rvert I$	1
	To prove LHS = RHS	1
	SECTION-II	
2. (a)	Finding slope	2
	Finding y-intercept	2
	OR	
	Writing general form	2
	Writing point-slope form	2
2 (b)	Writing formula	2
	Substitution	1
	Calculation/Simplification	2
	Final answer	1
	OR	
	Writing formula	2
	Substitution	1
	Calculation/Simplification	2
	Final answer	1

Q. No	Matter	Marks
2. (c)	Finding m_1 & m_2	2
	Writing Formula	1
	Substitution / Simplification	1
	Final answer	1
	OR	
	Formula	1
	Substitution	1
	Calculation / Simplification	2
	Final answer	1
2. (d)	Finding m_1	1
	Finding m_2	1
	Formula	1
	Substitution / Simplification	1
	Final answer	1
	OR	
	Finding m_1	1
	Finding m_2	1
	Writing Perpendicular Condition.	1
	Proof / Result	2
	SECTION - III	
3. (a)	degree to radian	2
	radian to degree	2
	OR	
	Splitting $75^\circ = 45^\circ + 30^\circ$	1
	Formula of $\cos(A+B)$	1
	Calculation	1
	Final answer	1.

Q. No.	Matter.	Marks
3. (b)	Writing $\sin(2A+A)$ formula Expansion	1
	$\sin 2A = 2\sin A \cos A$	1.
	$\cos 2A = 1 - 2\sin^2 A$	1
	$\cos^2 A = 1 - \sin^2 A$	1
	Simplification / Result	1.
	OR	
	$\tan(A+B)$ formula	1
	Finding $\tan(\pi/4) = 1$	1
	Simplication / Result	3
3. (c)	Finding $\sin 600^\circ$	1
	Finding $\cos 330^\circ$	1
	Finding $\cos 120^\circ$	1
	Finding $\sin 150^\circ$	1.
	Substitution / Simplification & Final answer	2
	OR	
	Finding $\cos(360-A)$	1
	Finding $\tan(360+A)$	1
	Finding $\cot(270-A)$	1
	Finding $\sin(90+A)$	1
	Substitution / Simplification & Final answer	2.

Q. No.	Matter	Marks
3. (d)	Expanding cos 20 cos 40	1
	Finding cos 80	1
	Simplification	2
	Final answer / Proof	1
	OR	
	Writing Formula	2
	Substitution / Simplification	2
	Final answer	1
	SECTION - IV	
4. (a)	Differentiation of x^3	1
	Differentiation of $5 \log x$	1
	Differentiation of $-2e^x$	1
	Differentiation of $\sin x$	1
	OR	
	Writing Product Rule	1
	Substitution	1
	Simplification / Result	2
4. (b)	Writing Quotient Rule	1
	Substitution	1
	Simplification	2
	Final answer	1
	OR	
	Finding dy/dx	1
	Finding d^2y/dx^2	1
	Substitution	1
	Simplification / Proof	2
4. (c)	Finding velocity	3
	Finding acceleration	3
	OR	
	Finding dy/dx	1
	Finding x values	1
	Finding minimum	2
	Finding maximum	2
4. (d)	Differentiating & finding Slope (m)	2
	Equation of tangent	1
	Substitution / Simplification	1
	Final answer	1
	OR	
	Finding dy/dx	1
	Finding d^2y/dx^2	1
	Substitution	1
	Simplification / Proof	2

Q. No.	Matter	Marks
5. (a)	Integrating e^x	1
	Integrating $1/x$	1
	Integrating $\cos x$	1
	Integrating x^3	1
	OR	
	Expanding $x(1+x^2)$	
	& Integrating x	2
	Integrating x^3	2
5. (b)	Formula	1
	Integrating (x^2+1)	2
	Applying limits	2
	Final answer	1
	OR	
	Substituting $\sin x = t$	1
	diff. $\sin x = t$	1
	Substitution $\int t^4 dt$	1
	Integration	2
	Again Substitution / Final answer	1
5. (c)	Formula	1
	Integrating x^2	1
	Applying limits	1
	Simplification/Result	2
	OR	
	Integration by Parts	2
	Substitution / Simplification	2
	Final answer	1
5 (d)	Integrating (x^2+1)	2
	Applying limits	2
	Final answer	1.
	OR	
	Formula: $\sin^3 x = \frac{3\sin x - \sin 3x}{4}$	1
	Integrating $\frac{3}{4}\sin x$	1
	Integrating $\frac{1}{4}\sin 3x$	1
	Applying limits	1
	Final answer	1.

I AND II SEMESTER EXAMINATION MAY/JUNE - 2024

ENGINEERING MATHEMATICS

CODE : 20SC01T

SECTION - I

1.

(a) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$A + A^T = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 5 & 8 \end{bmatrix}$

OR

$A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 1 \\ 3 & -2 \end{bmatrix}$

$A+B = \begin{bmatrix} 4 & 2 \\ 6 & 0 \end{bmatrix}$

(b) $A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$ $A^{-1} = \frac{1}{|A|} \text{adj} A$.

$|A| = \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = 4 - 3 = 1$.

$\text{adj} A = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$

$A^{-1} = \frac{1}{1}\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$

OR

$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$

$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 4 \\ 2 & 3-\lambda \end{vmatrix} = 0$

$(1-\lambda)(3-\lambda) - 8 = 0$

$\Rightarrow \lambda^2 - 4\lambda - 5 = 0$

$(\lambda - 5)(\lambda + 1) = 0$

$\boxed{\lambda = 5}$ $\boxed{\lambda = -1}$

(c) $4x - 3y = 5$ $3x + y = 7$

$\Delta = \begin{vmatrix} 4 & -3 \\ 3 & 1 \end{vmatrix} = 4 - (-9) = 13$

$\Delta_1 = \begin{vmatrix} 5 & -3 \\ 7 & 1 \end{vmatrix} = 5 - (-21) = 26$

$\Delta_2 = \begin{vmatrix} 4 & 5 \\ 3 & 7 \end{vmatrix} = 28 - 15 = 13$

$x = \frac{\Delta_1}{\Delta} = \frac{26}{13} = 2$; $y = \frac{\Delta_2}{\Delta} = \frac{13}{13} = 1$.

OR

$\begin{vmatrix} 1 & 2 & 1 \\ 3 & -1 & x \\ 2 & 3 & 2 \end{vmatrix} = 0$

$1(-2-3x) - 2(6-2x) + 1(9+2) = 0$

$-2 - 3x - 12 + 4x + 11 = 0$

$4x - 3x = 2 + 12 - 11$

$\boxed{x = 3}$

(d) $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$

$AB = \begin{bmatrix} 3+2 & 1+4 \\ 6+1 & 2+2 \end{bmatrix} = \begin{bmatrix} 5 & 5 \\ 7 & 4 \end{bmatrix}$

$(AB)^T = \begin{bmatrix} 5 & 7 \\ 5 & 4 \end{bmatrix}$

OR

$LHS = A(\text{adj} A) = \begin{bmatrix} 2 & 1 \\ 4 & 0 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ -4 & 2 \end{bmatrix}$

$= \begin{bmatrix} 0-4 & -2+2 \\ 0-0 & -4+0 \end{bmatrix}$

$= \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix} = -4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$|A| = (0-4) = -4$

$= |A| I = RHS$

SECTION – II

2.

(a) $2x+5y-3=0$

$ax+by+c=0$

$a=2,\ b=5,\ c=-3$

Slope, $m=\frac{-a}{b}=\frac{-2}{5}$

y-intercept, $=\frac{-c}{b}=\frac{-(-3)}{5}=\frac{3}{5}$

OR

(i) General form

$ax+by+c=0$

(ii) point-Slope form

$y-y_1=m(x-x_1)$

(b) $(2,-3)$ & $(1,0)$

x_1, y_1 $\quad x_2, y_2$

$y-y_1=\left(\frac{y_2-y_1}{x_2-x_1}\right)(x-x_1)$

$y-(-3)=\left(\frac{0-(-3)}{1-2}\right)(x-2)$

$y+3=\frac{3}{-1}(x-2)$

$-y-3=3x-6$

$3x-6+y+3=0$

$\boxed{3x+y-3=0}$

OR

x-intercept, $a=3$

y-intercept, $b=4$

$\frac{x}{a}+\frac{y}{b}=1$

$\frac{x}{3}+\frac{y}{4}=1$

$\frac{4x+3y}{12}=1$

$4x+3y=12$

$\boxed{4x+3y-12=0}$

(c) $3x+y+5=0$ & $2x+4y-7=0$

$m_1=\frac{-a}{b}=\frac{-3}{1}$ $\quad m_2=\frac{-a}{b}=\frac{-2}{4}=\frac{-1}{2}$

$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|=\left|\frac{-3+\frac{1}{2}}{1+(-3\times-\frac{1}{2})}\right|$

$\tan\theta=\left|\frac{\frac{-6+1}{2}}{1+\frac{3}{2}}\right|=\left|\frac{-5/2}{5/2}\right|=|-1|$

$\tan\theta=1$

$\Rightarrow \theta=\tan^{-1}1$

$\boxed{\theta=45^\circ}$

OR

point $=(1,3)$ Slope $m=2$

x_1, y_1

$y-y_1=m(x-x_1)$

$y-3=2(x-1)$

$y-3=2x-2$

$2x-2-y+3=0$

$\boxed{2x-y+1=0}$

OR

$\boxed{-2x+y-1=0}$

(d) Given, $5x+2y-3=0$

Slope $m_1 = \frac{-a}{b} = \frac{-5}{2}$

Parallel condition: $m_1 = m_2$

$\Rightarrow m_2 = \frac{-5}{2}$.

At $(3,2)$
$\quad\; x_1, y_1$

$\therefore \; y-y_1 = m_2(x-x_1)$

$y-2 = \frac{-5}{2}(x-3)$

$2y-4 = -5x+15$

$-5x+15-2y+4=0$

$\boxed{-5x-2y+19=0}$

or

$\boxed{5x+2y-19=0}$

OR

Given, $3x-2y+2=0 \quad -①$

Slope, $m_1 = \frac{-a}{b} = \frac{-3}{-2} = \frac{3}{2}$

$2x+3y+7=0$

Slope, $m_2 = \frac{-a}{b} = \frac{-2}{3}$

Perpendicular Condition

$m_1 m_2 = -1$

$\frac{\cancel{3}}{\cancel{2}} \times \frac{-\cancel{2}}{\cancel{3}} = -1$.

$-1 = -1$

LHS = RHS.

$\therefore$ Given lines are perpendicular to each other.

SECTION - III

3.

(a) (i) $120^\circ = \frac{120 \times \pi}{180} = \frac{2\pi}{3}$

(ii) $\frac{11\pi}{3} = \frac{11\cancel{\pi}}{3} \times \frac{180}{\cancel{\pi}} = 660^\circ$

OR

$\cos 75^\circ = \cos(45+30)$

$= \cos(A+B)$

$= \cos A \cos B - \sin A \cdot \sin B$

$= \cos 45 \cos 30 - \sin 45 \cdot \sin 30$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$\boxed{\cos 75^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}}$

(b) $\sin 3A = 3\sin A - 4\sin^3 A$

$\sin 3A = \sin(2A+A)$

$= \sin 2A \cos A + \cos 2A \sin A$

$= 2\sin A \cos A \cos A + (1-2\sin^2 A)\sin A$

$= 2\sin A \cos^2 A + \sin A - 2\sin^3 A$

$= 2\sin A(1-\sin^2 A) + \sin A - 2\sin^3 A$

$= 2\sin A - 2\sin^3 A + \sin A - 2\sin^3 A$

$\sin 3A = 3\sin A - 4\sin^3 A$.

OR

$A+B = \pi/4$

$\tan(A+B) = \tan \pi/4 = 1$.

$\frac{\tan A + \tan B}{1-\tan A \tan B} = 1$

$\tan A + \tan B = 1 - \tan A \tan B$

$1 + \tan A + \tan B + \tan A \tan B = 2$

$1 + \tan A + \tan B(1+\tan A) = 2$

$(1+\tan A)(1+\tan B) = 2$.

(c) $\sin 600^\circ \cos 330^\circ - \cos 120^\circ \sin 150^\circ$

$$\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$-\frac{3}{4} + \frac{1}{4} = \frac{-3+1}{4} = \frac{-2}{4} = \frac{-1}{2}$$

OR

$$\frac{\cos(360-A)\tan(360+A)}{\cot(270-A)\sin(90+A)}$$

$$\frac{\cos A \tan A}{\tan A \cos A} = 1$$

(d) $\cos 20 \cos 40 \cos 80$

$$= \frac{1}{2}\left[\cos(40+20) + \cos(40-20)\right]\cos 80$$

$$= \frac{1}{2}\left[\cos 60 + \cos 20\right]\cos 80$$

$$= \frac{1}{2}\left[\frac{1}{2} + \cos 20\right]\cos 80$$

$$= \left[\frac{1}{4} + \frac{1}{2}\cos 20\right]\cos 80$$

$$= \frac{1}{4}\cos 80 + \frac{1}{2}\cos 20 \cos 80$$

$$= \frac{1}{4}\cos(180-100) + \frac{1}{2}\times\frac{1}{2}\left[\cos 100 + \cos 60\right]$$

$$= \cancel{\frac{1}{4}(-\cos 100)} + \cancel{\frac{1}{4}[\cos 100]} + \frac{1}{4}\cos 60$$

$$= \frac{1}{4}\times\frac{1}{2} = \frac{1}{8}$$

4

OR

$$\tan A = \frac{1}{2} \quad \tan B = \frac{1}{3}$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2}\times\frac{1}{3}} = \frac{\frac{3+2}{6}}{1 - \frac{1}{6}}$$

$$= \frac{5/6}{5/6}$$

$$\boxed{\tan(A+B) = 1}$$

SECTION – IV

4.

(a) $y = x^3 + 5\log x - 2e^x + \sin x$

$$\frac{dy}{dx} = 3x^2 + 5\times\frac{1}{x} - 2e^x + \cos x$$

OR

$$\left\langle \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx} \right\rangle$$

$$y = x^2 \sin x.$$

$$\frac{dy}{dx} = x^2(\cos x) + (\sin x)(2x)$$

(b) $y = \dfrac{1+x}{1-x}$.

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\cdot\frac{du}{dx} - u\cdot\frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{(1-x)\frac{d}{dx}(1+x) - (1+x)\frac{d}{dx}(1-x)}{(1-x)^2}$$

$$= \frac{(1-x)(1) - (1+x)(-1)}{(1-x)^2}$$

$$= \frac{1-x+1+x}{(1-x)^2} = \frac{2}{(1-x)^2}$$

2

OR

$$y = \tan^{-1} x.$$

$$\frac{dy}{dx} = \frac{1}{1+x^2} \quad \& \quad \frac{d^2y}{dx^2} = \frac{-1}{1+x^2}(2x)$$

$$LHS = (1+x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx}$$

$$\cancel{(1+x^2)}\times\frac{-2x}{\cancel{(1+x^2)}} + 2x\cdot\frac{1}{(1+x^2)}$$

$$-2x + 2x = 0 = RHS$$

2

(c) $S = 2t^3 - t^2 + 5t - 3$

$v = \frac{ds}{dt} = 6t^2 - 2t + 5$

At $t = 1$ sec,

$v = 6(1)^2 - 2(1) + 5 = 6 - 2 + 5$

$\boxed{v = 9 \text{ m/s}}$

$v = 6t^2 - 2t + 5$

$a = \frac{dv}{dt} = 12t - 2$

At $t = 1$ sec,

$a = 12(1) - 2$

$\boxed{a = 10 \text{ m/s}^2}$

OR

$y = 2x^3 - 21x^2 + 36x + 50$

$\frac{dy}{dx} = 6x^2 - 42x + 36$

$\frac{dy}{dx} = x^2 - 7x + 6 = 0$

$\boxed{x = 6}$ & $\boxed{x = 1}$

$\frac{d^2y}{dx^2} = 2x - 7$

When $x = 6$, $\frac{d^2y}{dx^2} = 2(6) - 7 = 12 - 7$

$= 5 > 0$

$\boxed{\text{Minimum at } x = 6}$

$y = 2(6^3) - 21(6^2) + 36(6) + 50$

$\boxed{y_{min} = -58}$

When $x = 1$, $\frac{d^2y}{dx^2} = 2(1) - 7 = 2 - 7$

$= -5 < 0$

$\boxed{\text{Maximum at } x = 1}$

$y_{max} = 2(1^3) - 21(1^2) + 36(1) + 50$

$\boxed{y_{max} = 67}$

(d) $y = 2x^2 + x - 1$

$m = \frac{dy}{dx} = 4x + 1$

At $(1, 1)$ $m = 4(1) + 1 \Rightarrow \boxed{m = 5}$
x, y

Equation of tangent

At $(1, 1)$
x_1, y_1

$y - y_1 = m(x - x_1)$

$y - 1 = 5(x - 1)$

$y - 1 = 5x - 5$

$5x - 5 - y + 1 = 0$

$\boxed{5x - y - 4 = 0}$

OR

$y = \log x$

$\frac{dy}{dx} = \frac{1}{x}$ & $\frac{d^2y}{dx^2} = \frac{-1}{x^2}$

LHS $= x \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx}$

$= x \times \left(\frac{-1}{x^2}\right) + \frac{1}{x}$

$= -\frac{1}{x} + \frac{1}{x}$

$= 0$

$=$ RHS

SECTION - V

5.

(a) $y = e^x + \frac{1}{x} + \cos x + x^3$

$\int y\,dx = e^x + \log x + \sin x + \frac{x^4}{4}$

OR

$\int x(1+x^2)\,dx$

$\int (x + x^3)\,dx$

$\frac{x^2}{2} + \frac{x^4}{4} + c$

(b) $A = \int_a^b y\,dx$ Given $y = x^2+1$

$= \int_0^1 (x^2+1)\,dx$

$= \left[\frac{x^3}{3}\right]_0^1 + [x]_0^1$

$= \left(\frac{1^3}{3} - 0\right) + (1-0)$

$= \frac{1}{3} + 1 = \frac{1+3}{3} = \frac{4}{3}$ Sq. Units

OR

$y = \int \sin^4 x \cos x\,dx$

Put $\sin x = t$

$\cos x\,dx = dt$

$\therefore y = \int t^4 dt = \frac{t^5}{5} + c$

$y = \frac{(\sin x)^5}{5} + c$

(c) $V = \pi \int_a^b y^2 dx$ Given, $y^2 = x^2$

$V = \pi \int_1^2 x^2\,dx$

$= \pi \left[\frac{x^3}{3}\right]_1^2$

$= \pi \left[\frac{2^3}{3} - \frac{1^3}{3}\right]$

$= \pi \left[\frac{8}{3} - \frac{1}{3}\right] = \frac{7\pi}{3}$ Cubic Units

OR

$y = \int x e^x\,dx$

$\int uv'\,dx = uv - \int u'v'\,dx$

$\int x e^x dx \Rightarrow$ $u = x \Rightarrow u' = 1$

$\int v' = \int e^x dx$

$v = e^x$

$\therefore \int x e^x dx = x e^x - \int 1 e^x\,dx$

$= x e^x - e^x + C$

(d) $y = \int_0^1 (x^2+1)\,dx$

$\int y\,dx = \left[\frac{x^3}{3}\right]_0^1 + [x]_0^1$

$= \left(\frac{1^3}{3} - 0\right) + (1-0)$

$= \frac{1}{3} + 1 = \frac{1+3}{3} = \frac{4}{3}$

OR

$LHS = \int_0^{\pi/2} \sin^3 x\,dx = \int \frac{3\sin x - \sin 3x}{4}\,dx$

$= \frac{3}{4}\int_0^{\pi/2} \sin x\,dx - \frac{1}{4}\int_0^{\pi/2} \sin 3x\,dx$

$= -\frac{3}{4}\cos x \Big|_0^{\pi/2} + \frac{1}{4} \cdot \frac{\cos 3x}{3}\Big|_0^{\pi/2} + C$

$= -\frac{3}{4}(\cos \pi/2 - \cos 0) + \frac{1}{12}(\cos 3\pi/2 - \cos 0)$

$= -3/4\,(0-1) + 1/12\,(0-1) + C$

$= +3/4 - 1/12 = \frac{9-1}{12} = \frac{8}{12} = \frac{2}{3} = RHS$