VECTORS

$$\vec{a} = a_1 i + a_2 j + a_3 k = (a_1, a_2, a_3)$$
$$\vec{b} = b_1 i + b_2 j + b_3 k = (b_1, b_2, b_3)$$

modulus of a vector

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$
$$\vec{a} + \vec{b} = (a_1 + b_1) i + (a_2 + b_2) j + (a_3 + b_3) k$$

unit vector

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\vec{a}}{\sqrt{a_1^2 + a_2^2 + a_3^2}}$$

Dot product of two vectors

1. $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$
2. $\vec{a}.\vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$
3. *If two vectors are perpendicula or orthogonal then* $\vec{a}.\vec{b} = 0$
4. *projection of a vector on another vector*

   a) $\vec{a}$ *on* $\vec{b} = \frac{\vec{a}.\vec{b}}{\vec{b}}$

   b) $\vec{b}$ *on* $\vec{a} = \frac{\vec{a}.\vec{b}}{\vec{a}}$
5. *cosine of the angle between the two vectors*

$$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$$

6. *position vector of a point*

$$A = O\vec{A}$$
$$B = OB$$
$$C = OC$$
$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$
$$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$$
$$\overrightarrow{CA} = \overrightarrow{OA} - \overrightarrow{OC}$$

Cross product of two vectors

1. $\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$
2. *Area of a parallelogram*

$$|\vec{a} \times \vec{b}|$$

3. *Area of a triangle*

$$\frac{1}{2}|\vec{a} \times \vec{b}|$$

4. *sine of the angle between two vectors*

$$\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}$$

5. *Work Done*

$$w = \vec{F}.\vec{S}$$

where F is force

S is the displacement

6. *Moment of force*

$$m = \vec{r} \times \vec{F}$$

7. *unit vector perpendicular to both the vectors*

$$n = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$$

MATRICES

1. Definetions of matrices
2. Condition for adding and subtracting matrices the order must be same
3. Two matrices can be multiplied if and only if number of columns of first matrix must be equal to number of rows of second matrix
4. A^2 = A.A
   A^3=A.A.A
5. A=$(A^1)^1$
6. Determinant of a matrix is the single real value
7. Expansion of 2 X 2 matrix
   $$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$
   =product of principle diagonal elements – product of secondary diagonal elements.
8. Expansion of 3 X 3 matrix
   $$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$
   $$= a_1\begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2\begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3\begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$$
9. Crammer's rule
   $$x = \frac{\Delta_x}{\Delta},\ y = \frac{\Delta_y}{\Delta},\ z = \frac{\Delta_z}{\Delta}$$
10. $A^{-1} = \dfrac{adjA}{|A|}$
11. Characteristic equation $|A - \lambda I| = 0$
12. Cayley Hamilton theorem states ' Each and every square matrix satisfies its own characteristic equation.'
13. Steps to verify CHT
    a)find characteristic equation
    b)replace λ by matrix A
    c)LHS=RHS

LOGARITHMS

1. log(ab) = loga + logb
2. $\log\left(\dfrac{a}{b}\right)$= loga – logb
3. log a^m=mloga
4. $\log_a a = 1$
5. $\log_b a = \dfrac{\log a}{\log b}$
6. log 1 = 0

PROBABILITY

1. Events : A,B
2. Probability:P
3. Random variables: X,Y,Z
4. Probability of an event
   $$p(A) = \frac{n(E)}{n(s)}$$

S is the all possible outcomes
E is the any possible outcomes

5. Range of probability values
   $0 \le P(A) \le 1$
6. Certain event = 1
7. Impossible event = 0
8. Complement
   $$p(\overline{A}) = 1 - p(A)$$
9. General Addition rule for independent events
   $$p(A \cup B) = p(A) + p(B) - P(A \cap B)$$
10. Multiplication rule for independent events
    $$p(A \cap B) = p(A).p(B)$$
11. Conditional probability
    $$p(A/B) = \frac{p(A \cap B)}{p(B)}$$

TRIGONOMETRY

Trigonometric identities

1. $\sin^2\theta + \cos^2\theta = 1$
2. $1 + \tan^2\theta = \sec^2\theta$
3. $1 + \cot^2\theta = \cos ec^2\theta$
4. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
5. $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$
6. $\sin\theta.\cos ec\theta = 1$
7. $\cos\theta.\sec\theta = 1$
8. $\tan\theta.\cot\theta = 1$

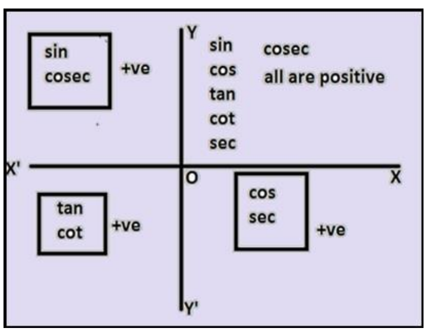
TRIGONOMETRIC RATIOS OF STANDARD ANGLES

	0^0	30^0	45^0	60^0	90^0
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

ALLIED ANGLES

Trigonometric ratios of negative angle

1. sin (-x) = -sin x
2. cos (-x) = cos x
3. tan (-x) = -tan x
4. csc (-x) = -csc x
5. sec (-x) = sec x
6. cot (-x) = -cot x



Steps for solving allied angle

Odd angle	Even angle
$90\pm\theta, 270\pm\theta$	$180\pm\theta,\ 360\pm\theta$

Le us solve the problem with steps

Find sin(120^0)

Step 1: splitting

sin(180-60) or sin(90+30)

step 2: grouping

sin(180-60) (even) or sin(90+30) (odd)

*if it is even no change of T function

*if it is odd the function changes to its co function

Sin ↔cos

sec↔cosec

tan↔cot

sin60 or cos30

step 3: quadranting

depending on which quadrant it lies we change the sign of the given problem

since the given problems lies under second quadrant therefore both are positive only.

Trig. ratio	-θ	90°-θ	90°+θ	180°-θ	180°+θ	360°-θ	360°+θ
sinθ	- sinθ	cosθ	cosθ	sinθ	- sinθ	- sinθ	sinθ
cosθ	cosθ	sinθ	- sinθ	- cosθ	- cosθ	cosθ	cosθ
tanθ	- tanθ	cotθ	- cotθ	-tanθ	tanθ	- tanθ	tanθ

Trig. ratio	-θ	90°-θ	90°+θ	180°-θ	180°+θ	360°-θ	360°+θ
cotθ	- cotθ	tanθ	-tanθ	-cotθ	cotθ	- cotθ	cotθ
secθ	secθ	cosecθ	- cosecθ	- secθ	- secθ	secθ	secθ
cosecθ	- cosecθ	secθ	secθ	cosecθ	-cosecθ	- cosecθ	cosecθ

COMPOUND ANGLES

1. $\sin(A+B)=\sin A\cos B+\cos A\sin B$
2. $\sin(A-B)=\sin A\cos B-\cos A\sin B$
3. $\cos(A+B)=\cos A\cos B-\sin A\sin B$
4. $\cos(A-B)=\cos A\cos B+\sin A\sin B$
5. $\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A.\tan B}$
6. $\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A.\tan B}$
7. $\sin 2A=2\sin A\cos A$
8. $\cos 2A=\cos^2 A-\sin^2 A$
9. $\cos 2A=2\cos^2 A-1$
10. $\cos 2A=1-2\sin^2 A$
11. $\tan 2A=\frac{2\tan A}{1-\tan^2 A}$
12. $\sin 3A=3\sin A-4\sin^3 A$
13. $\cos 3A=4\cos^3 A-3\cos A$
14. $\tan 3A=\frac{3\tan A-\tan^3 A}{1-3\tan^2 A}$

15. $\sin A = 2\sin\left(\frac{A}{2}\right)\cos\left(\frac{A}{2}\right)$

16. $\cos A = \cos^2\left(\frac{A}{2}\right) - \sin^2\left(\frac{A}{2}\right)$

17. $\tan A = \frac{2\tan\left(\frac{A}{2}\right)}{1-\tan^2\left(\frac{A}{2}\right)}$

TRANSFORMATION FORMULAE

1. $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$
2. $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$
3. $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$
4. $\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$
5. $\sin A\cos B = \frac{1}{2}[\sin(A+B)+\sin(A-B)]$
6. $\cos A\sin B = \frac{1}{2}[\sin(A+B)-\sin(A-B)]$
7. $\cos A\cos B = \frac{1}{2}[\cos(A+B)+\cos(A-B)]$
8. $\sin A\sin B = -\frac{1}{2}[\cos(A+B)-\cos(A-B)]$

LIMITS

Type 1: Direct substitution

Type 2 : Factorization

Type 3 : Rationalization

Type 4 : limit at ∞

Co efficient of highest variable will be the answer

Type 5 :

$\log_{x\to a}\frac{x^n-a^n}{x-a} = na^{n-1}$

Type 6:

$\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$

$lim_{\theta\to 0}\frac{\tan\theta}{\theta} = 1$

COMPLEX NUMBERS

C = $\{\ a+ib \ / \ a,b\in R,\ i=\sqrt{-1}\ \}$

a is the real part

b is the imaginary part

$i^1 = i$	$i^5 = i$	$i^{4n+1} = i$
$i^2 = -1$	$i^6 = -1$	$i^{4n+2} = -1$
$i^3 = -i$	$i^7 = -i$	$i^{4n+3} = -i$
$i^4 = 1$	$i^8 = 1$	$i^{4n} = 1$

Addition/subtraction of complex numbers

$(a\pm ib)+(c\pm id) = (a\pm c)+i(b\pm d)$

Multiplication of two complex numbers

$(a+ib).(c+id) = ac+iad+ibc+i^2bd$

$= ac+iad+ib-bd$

$= (ac-bd)+i(ad+bc)$

Conjugate of a complex number

Z=a+ib	Z=a-ib
1+i	1-i
i	-i
1+i $\sqrt{3}$	1-i $\sqrt{3}$
-1+i $\sqrt{3}$	-1-i $\sqrt{3}$
1	1

Modulus of a complex number

If z = x + iy

Then modulus is denoted by

r = $\sqrt{x^2+y^2}$

amplitude of a complex number is denoted by

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

It varies in different quadrants if $\alpha = \tan^{-1}\left(\frac{y}{x}\right)$

	x	y	θ
I	+	+	α
II	-	+	$\pi-\alpha$
III	-	-	$-(\pi-\alpha)$
IV	+	-	- α

POLAR FORM OF A COMPLEX NUMBER

if z = x+iy is the Cartesian form of a complex number then polar form of a complex is defined as

Z = r(cos θ +isin θ) where

r is the modulus

θ is the amplitude of a complex number

ARGAND DIAGRAM OF A COMPLEX NUMBER

To draw the argand diagram just draw a circle of any radius and mark the angle made by its amplitude