

1.  $\frac{dk}{dx} = 0$

2.  $x = 1$

3.  $x^2 = 2x$

4.  $x^3 = 3x^2$

5.  $x^n = n x^{n-1}$

6.  $\frac{1}{x^n} = \frac{-n}{x^{n+1}}$

7.  $\frac{1}{x} = -\frac{1}{x^2}$

8.  $\frac{1}{x^2} = -\frac{2}{x^3}$

9.  $\frac{1}{x^3} = -\frac{3}{x^4}$

10.  $\log x = \frac{1}{x}$

11.  $\sqrt{x} = \frac{1}{2\sqrt{x}}$

12.  $e^{ax} = ae^{ax}$

13.  $e^x = e^x$

14.  $e^{ax} = ae^{ax}$

15.  $e^{-x} = -e^{-x}$

16.  $e^{-2x} = -2e^{-2x}$

17.  $e^{3x} = 3e^{3x}$

18.  $a^x = a^x \log a$

19.  $2^x = 2^x \log 2$

20.  $3^x = 3^x \log 3$

21.  $\frac{d(\sin ax)}{dx} = a \cos ax$

22.  $\frac{d(\sin x)}{dx} = \cos x$

23.  $\frac{d(\cos ax)}{dx} = -a \sin ax$

24.  $\frac{d(\cos x)}{dx} = -\sin x$

25.  $\frac{d(\tan ax)}{dx} = a \sec^2 x$

26.  $\frac{d(\tan x)}{dx} = \sec^2 x$

27.  $\frac{d(\cos ecx)}{dx} = -a \cos ecx \cot ax$

28.  $\frac{d(\cos ecx)}{dx} = -\cos ecx \cot x$

29.  $\frac{d(\sec ax)}{dx} = a \sec ax \tan ax$

30.  $\frac{d(\sec x)}{dx} = \sec x \tan x$

31.  $\frac{d(\cot ax)}{dx} = -a \cos ec^2 ax$

32.  $\frac{d(\cot x)}{dx} = -\cos ec^2 x$

**RULES OF DIFFERENTIATION**

**1. ALGEBRAIC RULE**

$y = u + v$

$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

**2. PRODUCT RULE**

$y = uv$

$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$

$\frac{dy}{dx} = uv^1 + vu^1$

**3. QUOTIENT RULE**

$y = \frac{u}{v}$

$\frac{dy}{dx} = \frac{vu^1 - uv^1}{v^2}$

**4. CHAIN RULE**

1.  $\frac{d(f(x))^n}{dx} = n(f(x))^{n-1} f^1(x)$

2.  $\frac{d(\sqrt{f(x)})}{dx} = \frac{1}{2\sqrt{f(x)}} f^1(x)$

3.  $\frac{d(\log f(x))}{dx} = \frac{1}{f(x)} f^1(x)$

4.  $\frac{d(\sin(f(x)))}{dx} = \cos(f(x)) f^1(x)$

**MAIN PROBLEMS ON CHAIN RULE**

1.  $\frac{d(\sin^2 x)}{dx} = 2 \sin x \cos x$

2.  $\frac{d(\cos^2 x)}{dx} = 2 \cos x (-\sin x)$

3.  $\frac{d(\tan^2 x)}{dx} = 2 \tan x (\sec^2 x)$

$$4. \frac{d(\sin^3 x)}{dx} = 3\sin^2 x \cdot \cos x$$

$$5. \frac{d(\cos^3 x)}{dx} = 3\cos^2 x(-\sin x)$$

$$6. \frac{d(\tan^3 x)}{dx} = 3\tan^2 x(\sec^2 x)$$

$$7. \frac{d(\sqrt{\sin x})}{dx} = \frac{1}{2\sqrt{\sin x}} \cos x$$

### IMPLICIT FUNCTIONS

$$1. y = y^1$$

$$2. y^2 = 2yy^1$$

$$3. y^3 = 3y^2 y^1$$

$$4. y^4 = 4y^3 y^1$$

$$5. xy = (xy^1 + y)$$

$$6. x^2 y = (x^2 y^1 + y(2x))$$

$$7. x^2 y^2 = x^2(2yy^1) + y^2(2x)$$

$$8. 3xy = 3(xy^1 + y)$$

$$9. 3x^2 y = 3(x^2 y^1 + y(2x))$$

$$10. \sin y = \cos y \cdot y^1$$

### PARAMETRIC DIFFERENTIATION

$$x = f(t), \quad y = g(t)$$

then

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$1. x = at, \quad y = \frac{a}{t}$$

$$2. x = a \sin^2 t, \quad y = b \cos^2 t$$

$$3. x = a \sin^3 t, \quad y = b \cos^3 t$$

$$4. x = a(\sin \theta - \cos \theta), \\ y = b(\theta + \sin \theta)$$

### LOGARITHMIC DIFFERENTIATION

$$\log a^m = m \log a$$

$$\log mn = \log m + \log n$$

$$y = f(x)^{g(x)}$$

applying log on both sides

$$\log y = \log f(x)^{g(x)}$$

$$\log y = g(x) \log f(x)$$

$$y = x^x, \quad y = \sin x^{\cos x}, \quad y = \cos x^{\sin x}$$

$$y = \tan x^{\sin x}, \quad y = \log x^{\sin x}, \quad y = \sin x^{\log x}$$

### SUCCESSIVE DIFFERENTIATION

Differentiating twice

### APPLICATION OF DIFFERENTIATION

#### 1. Formula for slope of a tangent

$$m = \frac{dy}{dx} = \tan \theta$$

#### 2. Slope of a normal

$$N = -\frac{1}{m} = -\frac{1}{\frac{dy}{dx}}$$

#### 3. Equation of a tangent

$$y - y_1 = m(x - x_1)$$

#### 4. Equation of normal

$$y - y_1 = \frac{1}{-m}(x - x_1)$$

#### 5. Velocity

$$v = \frac{dy}{dx}$$

#### 6. Acceleration

$$a = \frac{d^2 y}{dx^2} = \text{differentiating twice}$$

$$7. \text{Area of circle, } A = \pi r^2, \quad \frac{dA}{dt} = 2\pi r,$$

#### 8. Volume of a sphere,

$$V = \frac{4}{3}\pi r^3, \quad \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$9. \text{Area of square, } A = a \times a$$

$$10. \text{Area of rectangle, } A = l \times b$$

# ENGG MATHS-II FORMULAE AND STUDY PACKAGE

## DIFFERENTIATION

1. Differentiate the following w.r.t.  $x$  by the method of first principal

1.  $x^n$  2.  $\sin x$  3.  $\cos x$  4.  $\tan x$

2. Differentiate the following

1.  $x^{-100}$  2.  $\sqrt{x}$  3.  $e^{2x}$  4.  $x^{29}$

$\tan x$   $\log x$   $5^x$   $9^x$

$100^x$   $x^n$   $x^n + \frac{1}{x^n}$   $\operatorname{cosec} x$

$\sec x$   $\frac{1}{x}$   $\cos ax$   $\tan ax$

$x^3$   $x^2$   $x$   $1$

$5$   $100$   $3$   $9$

$2x$   $5x$   $6x$   $7x$

$-x$   $-2x$   $-4x$   $-\log x$

$e^{-2x}$   $e^{-9x}$   $e^{-x}$   $e^{-3x}$

$\sin ax$   $\tan 3x$   $\cos 4x$   $\tan 6x$

$\sec 4x$   $-\tan x$   $-\sin x$   $-\cos x$

$x^{-n}$   $5\sin x$   $4e^{-2x}$   $5x^2$

$6x^3$   $9x$   $9$   $5\log x$

$\tan 6x$   $\sin 4x$   $\tan 10x$   $\operatorname{cosec} 4x$

$0$   $-2x$   $-\frac{1}{x}$   $-x$

3. Differentiate the following

$\sqrt{x} + \log x + \sin^{-1} x + e^{\tan x} - a^x$

$6x^3 - 3\cos x + 4\cot x + 2e^x - \frac{5}{x}$

$x^3 + \sec x - \log x + e^{-3x} - \sqrt{x}$

$ax^3 + bx^2 + cx + d - \frac{1}{x} - \operatorname{cosec} x$

$\log x + x^4 + \sec x - \cos ex - e^{-2x}$

$\tan x + x^5 + \log x + 5x + 9$

$\tan x + x^7 + e^{\tan x} + \log x + 5$

4. **PRODUCT RULE**

$x \tan x$   $x^3 \log x$   $e^x \sin x$   $xe^x$

$x^3 \sin x$   $\cot x \cdot 9^x$   $x^3 3^x$   $4x \operatorname{cosec} x$

$\sqrt{x} \tan x$   $e^{3x} \sqrt{x}$   $4^x e^{2x}$   $x^{-4} \sec 9x$

5. **QUOTIENT RULE**

$\frac{\cos x}{x}$   $\frac{\tan x}{x}$   $\frac{a \sin x + b}{a + b \cos x}$

$\frac{x^3 + x - 3}{x^2 - 5x + 1}$   $\frac{\cos x}{2 + \sin x}$   $\frac{x \log x}{x - 1}$

$\frac{1 - \sin x}{1 + \sin x}$   $\frac{\cos x + \sin x}{\cos x - \sin x}$   $\frac{1 - e^x}{1 + e^x}$

$\frac{x + 3}{x^2 - 1}$   $\frac{\sin x}{1 + \cos x}$   $\frac{1 + x + x^2}{1 - x + x^2}$

6. **CHAIN RULE**

$\sqrt{x^2 + 4x + 6}$   $\cos ax$   $\cos^2(3x - 7)$

$\log(\sec x + \tan x)$   $\log \sqrt{\sin(\sqrt{x})}$   $\sqrt{\cos x}$

$\sqrt{\frac{1 - \cos x}{1 + \cos x}}$   $\sqrt{1 + \cos^2 x}$   $\sin^3 x$

$\sqrt{x \cos x}$   $(x + 1)^2 \tan \sqrt{x}$   $e^{\sin^{-1} x}$

7. **IMPLICIT FUNCTIONS**

$y^2, y^3, y^5, y$

$x^2 + y^2 = a^2$

$y = x^2 + \sqrt{y} + xy$

$2x^2 - 3xy + 4y^2 = 1$

$x^2 + y^2 + 2x + 2y = 2xy$

$\log x + \log y = x^3 + y^3$

$x^2(x - y) = 4$

$x^3 = 1 + \sqrt{y}$

$x^3 + y^3 = 3axy$

If  $x^2 + xy + y^2 = 0$  evaluate  $\frac{dy}{dx}$  at  $x = 1, y = 2$

$ax^2 + 2hxy + by^2 + 8 = 0$

$x^2 + xy + y^2 = 0$

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

$\sqrt{x} + 2xy + \sqrt{y} + 4 = 0$

if  $x^2 + y^2 = xy$  then prove that  $\frac{dy}{dx} = \frac{y - 2x}{2x - y}$

8. **PARAMETRIC FUNCTION**

$x = a\theta^2$   $y = 2a\theta$

$x = a \cos^3 \theta$   $y = b \sin^3 \theta$

$x = a \cos \theta$   $y = a \sin \theta$

$x = a(\cos \theta + a \sin \theta)$   $y = a(\sin \theta - \theta \cos \theta)$

$x = 2 \sin \theta$   $y = \cos 2\theta$  at  $\theta = \frac{\pi}{4}$

$x = a \cos^4 t$   $y = b \sin^4 t$

$x = a \cos \theta$   $y = b \sin \theta$  at  $\theta = \frac{\pi}{4}$

9. **LOGARITHMIC FUNCTION**

$y = x^x, y = x^{\frac{1}{x}}, y = \sin x^{\cos x}, y = \tan x^{\cos x}$

$y = \log x^{\log x}, y = x^{\log x}, y = \cos x^{\sin x}$

10. **Differentiation of a function w.r.t. another function**

Differentiate  $\sin x$  w.r.t.  $\cos x$

Differentiate  $x^3$  w.r.t.  $\tan^{-1} x$

Differentiate  $\cos x$  w.r.t.  $e^x$

Differentiate  $e^{\sqrt{\sin x}}$  w.r.t.  $x^2$

## 11. Successive differentiation

Find the second order derivative of the following functions

$$(ax+b)^7$$

$$\log(ax+b)$$

$$ax^3+bx^2+cx+d$$

$$\sin^2 x$$

1. If  $y = xe^x$  find  $\frac{d^2y}{dx^2}$  at  $x=0$

2. if  $y = ae^{mx} + be^{-mx}$  then prove that

$$\frac{d^2y}{dx^2} - m^2y = 0$$

3. If  $y = (\tan^{-1}x)^2$  then prove that

$$(1+x^2)^2 y_2 + 2x(1+x^2)y_1 - 2 = 0$$

4. If  $y = \tan^{-1}x$  prove that  $(1+x^2)y_2 - 12xy_1 = 0$

5. If  $y = e^{\tan^{-1}x}$  TPT  $(1+x^2)y_2 + (2x-1)y_1 = 0$

6. If  $y = \tan^{-1}x$  TPT  $(1+x^2)y_2 + 2xy_1 = 0$

7. if  $y = e^{3x}$  find  $\frac{d^2y}{dx^2}$  at  $x=0$

8. If  $y = e^x \sin x$  find  $\frac{d^2y}{dx^2}$

### APPLICATION OF DERIVATIVES

1. Find the equation of tangent to the curve

$$x^2 + 2y^2 = 4 \text{ at } (2, -1) \text{ on it.}$$

2. Find the equation of tangent to the curve

$$y = 3x^2 - 6x \text{ at } (2, 1) \text{ on it.}$$

3. Find the equation of tangent to the curve

$$y = x^2 + x + 4 \text{ at the point where it crosses y-axis}$$

4. Find the equation of tangent and normal to the

$$\text{curve } y = x^3 + 4x^2 - 6x + 5 \text{ at } (1, -4) \text{ on it.}$$

1. Find the points on the curve  $y = 2x^3 - 3x^2 + 5$

where tangent is parallel to x-axis

2. Find the equation of tangent and normal to the

$$\text{curve } y(x^2 + 1) = x + 3 \text{ at } (2, 1) \text{ on it.}$$

3. Find the equation of normal to the curve

$$y^2 = 16x \text{ at } \left(\frac{1}{2}, 1\right) \text{ on it.}$$

4. Find the slope tangent to the curve

$$x^2 + 2y^2 = 9 \text{ at } (1, 2) \text{ on it.}$$

5. find the slope of the normal to the curve

$$y = 2x^2 + x + 6 \text{ at } (1, -1) \text{ on it.}$$

6. find the slope of the tangent to the curve

$$y = 2x^2 + 1 \text{ at } (0, 1) \text{ on it.}$$

7. Find the slope tangent to the curve

$$y = 2x^2 + 4x - 3 \text{ at } (2, 3) \text{ on it.}$$

8. Find the slope tangent to the curve  $3x^2 + 2y^2 = 5$  at  $(1, 1)$  on it.

9. Find the slope tangent to the curve

$$y = 2x^2 - 3x - 1 \text{ at } (1, -2) \text{ on it.}$$

### DERIVATIVE AS A RATE MEASURE

1. The equation of motion is given by  $s = 2t^2 - 3t + 1$ . find the velocity after 3 sec

2. The equation of motion is given by  $s = (2t + 1)^2$ . find the velocity and acceleration

3. The equation of motion is given by  $s = t^3 + t^2 - 3t + 6$ . find the velocity and acceleration after 2 sec

4. The equation of motion is given by  $s = 4t^3 - 6t^2 + t - 7$ . find the velocity after 2 sec

5. The equation of motion is given by  $s = t^3 + 3t + 5$ . find the velocity after 3 sec

6. The equation of motion is given by  $S = 4t^3 - 6t^2 + t - 7$ . find the velocity after 2 sec

7. The equation of motion is given by  $s = 4t^3 - 6t^2 + t - 7$ . find the velocity and acceleration after 2 sec

8. A ball is thrown vertically upwards and reaches maximum height S in metre. the height reached is given by  $S = -16t^2 + 64t$ . how much time is taken to reach maximum height?

9. Distance travelled by a car is given by  $S = 160t - 16t^2$  metre and time in sec. when does the car stop?

10. The circular patch of oil spreads on water. the area is increasing at the rate  $16\text{cm}^2/\text{sec}$ . find the rate of increasing of radius when the radius is 12cm

11. The volume of a spherical ball is increasing at the rate of  $36\pi\text{cc}/\text{sec}$ . Find the rate at which the radius is increasing when the radius of the ball is 2cm.

12. The volume of a sphere is increasing at the rate of  $2\text{cc}/\text{sec}$ . Find the rate at which the radius is increasing when the radius of the ball is 15cm.

13. If the volume of a balloon which is being inflated at the rate of  $6\text{cc}/\text{min}$ . Find the rate at which its surface area is increasing when the radius of the balloon is 12cm.

14. The radius of a spherical balloon is increasing at the rate of  $2\text{cm}/\text{s}$ . Find the rate of increase of volume when the radius is 6cm.

15. Water is flowing into a right circular cylindrical tank of radius 50cm. At the rate of  $500\text{cc}/\text{min}$ . find how fast is the level of water going up

16. Show that the curve  $2x^3 - y = 0$  is increasing at the point  $(1, 2)$

17. Find maximum and minimum values of the function  $x^3 - 6x^2 + 9x + 7$

18. Find maximum and minimum values of the function  $x^3 + 6x^2 - 15x + 5$

19. Find maximum and minimum values of the function  $x^3 + 6x^2 + 9x$

20. Find maximum and minimum values of the function  $x^3 - 3x + 4$