$+ \quad - \quad +$ **SE(**

(a) Solve for x, $\begin{vmatrix} 1 & 2 & 3 \\ 2 & x & 3 \\ 3 & 4 & 3 \end{vmatrix} = 0$

$$1(3x-12) - 2(6-9) + 3(8-3x) = 0$$

S1

S2

S4

$$3x - 12 + 6 + 24 - 9x = 0$$

$$-6x + 18 = 0$$

$$\Rightarrow \cancel{-}6x = \cancel{-}18$$

$$\Rightarrow \quad x = \frac{\cancel{18}^{\,3}}{\cancel{6}}$$

$$\boxed{x = 3}$$

OR

If $A = \begin{bmatrix} 1 & 5 \\ 2 & 3 \end{bmatrix}$, Find $A + A^T$.

$$A + A^T = \begin{bmatrix} 1 & 5 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 5 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 7 \\ 7 & 6 \end{bmatrix} \quad //$$

(b) Using Cramer's rule, find the solution of the system of equations $2y - z = 0$, and $x + 3y = -4$, $3x + 4y = 4$

$$A = \begin{bmatrix} 0 & 2 & -1 \\ 1 & 3 & 0 \\ 3 & 4 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 \\ -4 \\ 4 \end{bmatrix}$$

$$\begin{array}{ccc} + & - & + \end{array}$$

$$\Delta = \begin{vmatrix} 0 & 2 & -1 \\ 1 & 3 & 0 \\ 3 & 4 & 0 \end{vmatrix} = 0 - 2(0-0) - 1(4-9)$$

$$= +5$$

$$\begin{array}{ccc} + & - & + \end{array}$$

$$\Delta_x = \begin{vmatrix} 0 & 2 & -1 \\ -4 & 3 & 0 \\ +4 & 4 & 0 \end{vmatrix} = 0 - 2(0-0) - 1(-16+12)$$

$$= +28$$

$$\Delta_y = \begin{vmatrix} 0 & 0 & -1 \\ 1 & -4 & 0 \\ 3 & 4 & 0 \end{vmatrix} = 0 - 0 - 1(4+12)$$

$$= -16$$

$$\Delta_z = \begin{vmatrix} 0 & 2 & 0 \\ 1 & 3 & -4 \\ 3 & 4 & 4 \end{vmatrix} = 0 - 2(4+12) + 0$$

$$= -32$$

$$\boxed{x = \frac{\Delta_x}{\Delta} = \frac{28}{5}}, \quad \boxed{y = \frac{\Delta_y}{\Delta} = \frac{-16}{5}}, \quad \boxed{z = \frac{\Delta_z}{\Delta} = \frac{-32}{5}}$$

Which of the matrix has no inverse ?

$$A = \begin{bmatrix} 1 & 5 \\ 0 & -1 \end{bmatrix}, B = \begin{bmatrix} 2 & 6 \\ -1 & -3 \end{bmatrix} C = \begin{bmatrix} 3 & 2 \\ 12 & 8 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 5 \\ 0 & -1 \end{vmatrix} = -1 - 0 = -1$$

$$|B| = \begin{vmatrix} 2 & 6 \\ -1 & -3 \end{vmatrix} = -6 + 6 = 0$$

$$|C| = \begin{vmatrix} 3 & 2 \\ 12 & 8 \end{vmatrix} = \cancel{24} - \cancel{24} = 0$$

For B & C A^{-1} doesn't exists.

(c) Find the characteristic equation and characteristic roots value for the matrix **5**

$$\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\lambda^2 - (Tr\,A)\lambda + \Delta = 0$$

$$Tr = 2, \quad \Delta = 1 - 6 = -5$$

$$\boxed{\lambda^2 - 2\lambda - 5 = 0}$$

$$x_1 = 1 + \sqrt{6}, \quad x_2 = 1 - \sqrt{6}$$

$$\boxed{\lambda_1 = 3.44, \quad \lambda_2 = -1.44}$$

OR

If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ 4 & 3 \end{bmatrix}$, then verify that $(A+B)^T = A^T + B^T$

$$A+B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 7 & 7 \end{bmatrix}$$

$$(A+B)^T = \begin{bmatrix} 3 & 7 \\ 2 & 7 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \quad B^T = \begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix}$$

$$A^T + B^T = \begin{bmatrix} 3 & 7 \\ 2 & 7 \end{bmatrix} \Rightarrow (A+B)^T = A^T + B^T$$

If $A = \begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}$, find A^{-1}.

$$A^{-1} = \frac{\text{adj}\,A}{|A|}$$

$$\text{adj}\,A = \begin{bmatrix} 5 & -1 \\ -3 & 2 \end{bmatrix} \qquad |A| = 10 - 3 = 7$$

$$\therefore \quad A^{-1} = \frac{\begin{bmatrix} 5 & -1 \\ -3 & 2 \end{bmatrix}}{7}$$

OR

If $A = \begin{bmatrix} 2 & 1 \\ -3 & 5 \end{bmatrix}$ & $B = \begin{bmatrix} 9 & 2 \\ -3 & 5 \end{bmatrix}$, find AB.

$$AB = \begin{bmatrix} 2 & 1 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} 9 & 2 \\ -3 & 5 \end{bmatrix} = \begin{bmatrix} 18-3 & 4+5 \\ -27-15 & -6+25 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 9 \\ -42 & -31 \end{bmatrix}$$

(Match the following)

P

2. (a) (A) Equation of a straight line passing through a given point (x, y) and having slope m is

(B) Equation of a straight line passing through two points (x_1, y_1) and (x_2, y_2) is

(C) The equation of a straight line whose x and y-intercepts are a, b respectively is

(D) If two lines are perpendicular then product of their slopes is equal to

Q

(1) $\frac{x}{a} + \frac{y}{b} = 1$

(2) $y - y_1 = m(x - x_1)$

(3) -1

(4) $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$ **4**

Answers :

P	Q
A	2
B	4
C	1
D	3

2

4

1

3

P

(A) If two lines with slopes m_1 and m_2 are parallel then 'θ' is

(B) Equation of a straight line whose slope is m and y intercept is C.

(C) Slope of line $ax + by + c = 0$

(D) Slope of a line joining two points $(x_1\ y_1)$ and $(x_2\ y_2)$.

Q

(1) $y = mx + c$

(2) 0 (zero)

(3) $\dfrac{y_2 - y_1}{x_2 - x_1}$

(4) $-\dfrac{a}{b}$

Answers :

P	Q
A	2
B	1
C	4
D	3

$\Rightarrow$

(b) Find the equation to the perpendicular to the line $6x - 5y - 2 = 0$ and passing through $(2, -3)$. **6**

OR

$$bx - ay + k = 0$$

$$a = 6, \quad b = -5, \quad c = -2$$

$$-5x - 6y + k = 0$$

$$-5(2) - 6(-3) + k = 0$$

$$-10 + 18 + k = 0$$

$$8 + k = 0 \Rightarrow k = -8$$

$$-5x - 6y - 8 = 0 \Rightarrow \boxed{5x + 6y + 8 = 0}$$

OR

Are the lines 4x + 6y + 7 =0 and 2x + 3y – 1 = 0 parallel to each other? Justify.

$4x + 6y + 7 = 0$

$m_1 = -\frac{\cancel{4}\,2}{\cancel{6}\,3} = -\frac{a}{b}$

$= \frac{-2}{3}$

$2x + 3y - 1 = 0$

$m_2 = -\frac{a}{b} = \frac{-2}{3}$

Since, $\boxed{m_1 = m_2}$

lines $= \parallel$

(c) Find the equation of straight line parallel to $5x + 6y - 10 = 0$ and passing through the point $(-3, 3)$. **5**

OR

$$5x + 6y - 10 = 0$$

$$5x + 6y + k = 0$$

$$5(-3) + 6(3) + k = 0$$

$$-15 + 18 + k = 0$$

$$3 + k = 0$$

$$k = -3$$

$$\boxed{5x + 6y - 3 = 0}$$

Are the lines $3x + 4y + 7 = 0$ and $28x - 21y + 50 = 0$ are perpendicular to each other ? Justify.

$m_1 = -\frac{a}{b} = -\frac{3}{4}$, $m_2 = -\frac{a}{b} = -\frac{\cancel{28}^{4}}{+\cancel{21}_{3}}$

$= +\frac{4}{3}$

$m_1 \cdot m_2 = -\frac{3}{\cancel{4}} \times \frac{\cancel{4}}{3} = -1$

Since, $m_1 \cdot m_2 = -1$

$\therefore l_1 \perp l_2$

Find the angle between the lines $x + 3y + 5 = 0$ and $4x + 2y - 7 = 0$ **5**

$$m_1 = -\frac{1}{3}, \qquad m_2 = -\frac{\cancel{4}^{\,2}}{\cancel{2}} = -2$$

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \left|\frac{-2 + \frac{1}{3}}{1 + \left(-\frac{1}{3}\right)(-2)}\right|$$

$$= \left|\frac{-\frac{5}{\cancel{3}}}{\frac{5}{\cancel{3}}}\right| = 1$$

$$\Rightarrow \tan\theta = 1 \Rightarrow \boxed{\theta = 45^\circ}$$

Find the equation of straight line which passes through the points (–2, 3) and (–5, 6).

$$\Rightarrow (x_1, y_1) = (-2, 3)$$
$$(x_2, y_2) = (-5, 6)$$

$$\Rightarrow \frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{y-3}{x+2} = \frac{\cancel{3}}{-\cancel{3}}$$

$$\frac{y-3}{x+2} = -1 \Rightarrow y - 3 = -1(x+2)$$

$$y - 3 = -x - 2$$

$$\boxed{x + y - 1 = 0}$$

Determine the value of cos(570°) and sin(330°).

OR

$$\cos(570) = \cos(360+210)$$
$$= \cos 210$$
$$= \cos(180+30) = -\cos 30 = -\frac{\sqrt{3}}{2}$$

$$\sin 330 = \sin(360-30)$$
$$= -\sin 30 = -\frac{1}{2}$$

Convert 45 degree into radian and $\frac{11\pi}{5}$ radian into degree.

$$45^\circ = \cancel{45} \times \frac{\pi}{\cancel{180}_{4}} = \frac{\pi}{4}$$

$$\frac{11\pi}{5} = \frac{11 \times \overset{36}{\cancel{180}}}{\cancel{5}} = 396$$

If $A + B = \frac{\pi}{4}$ prove that $(1 + \tan A)(1 + \tan B) = 2$

$$A + B = \frac{\pi}{4}$$

$$\tan(A+B) = \tan\frac{\pi}{4}$$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = 1$$

$$\tan A + \tan B = 1 - \tan A \tan B$$

adding 1 obs

$$1 + \tan A + \tan B = 2 - \tan A \tan B$$

$$1 + \tan A + \tan B + \tan A \tan B = 2 \Rightarrow (1+\tan A)(1+\tan B) = 2$$

OR

Prove that $\sin 3A = 3\sin A - 4\sin^3 A$

$$\sin(2A + A) = \sin 2A \cos A + \cos 2A \sin A$$

$$= 2\sin A \cos A + (1 - 2\sin^2 A)\sin A$$

Given $\tan A = \frac{18}{17}$ and $\tan B = \frac{1}{35}$ show that $A - B = \frac{\pi}{4}$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$= \frac{\frac{18}{17} - \frac{1}{35}}{1 + \frac{18}{17} \times \frac{1}{35}} = \frac{\frac{263}{595}}{1 + \frac{18}{595}}$$

$$= \qquad = \frac{\frac{263}{\cancel{595}}}{\frac{613}{\cancel{595}}}$$

Show that : $\dfrac{\cos(360° - A) \cdot \tan(360° + A)}{\cot(270° - A) \cdot \sin(90° + A)} = 1$

$$\text{LHS} = \frac{\cos(360 - A) \; \tan(360 + A)}{\cot(270 - A) \cdot \sin(90 + A)}$$

$$= \frac{(+\cancel{\cos A})\,(+\cancel{\tan A})}{(+\cancel{\tan A})\,(+\cancel{\cos A})} = 1$$

Prove that $\cos 55° + \cos 65° + \cos 175° = 0$

$$= \cos(175) + \cos 65 + \cos 55$$

$$= \cos 175 + 2\cos 60 \times \cos 5$$

$$= \cos 175 + \cancel{2} \times \frac{1}{\cancel{2}} \times \cos 5$$

$$=$$

Show that $\dfrac{\sin 40^\circ + \sin 20^\circ}{\cos 40^\circ + \cos 20^\circ} = \dfrac{1}{\sqrt{3}}$

$$= \frac{\cancel{2} \sin(30) \cancel{\cos(10)}}{\cancel{2} \cos(30) \cancel{\cos(10)}} = \tan 30 = \frac{1}{\sqrt{3}}$$

If $y = \sin x + \log x + e^x + \tan x$, then find $\frac{dy}{dx} = ?$

$$y' = \cos x + \frac{1}{x} + e^x + \sec^2 x$$

If $\frac{dy}{dx} = 4x^3 + 3x^2$, then find $\frac{d^2y}{dx^2}$ at (1, 2)

$$y' = 12x^2 + 6x$$

$$y'' = 24x + 6$$

$$y''\Big|_{(1,2)} = 24(1) + 6 = 30 \;//$$

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Using chain rule of differentiation, find the derivative of the function $y = (3x + 8)^5$ 6

OR

$$\boxed{y' = 5(3x+8)^4(3)}$$

OR

Using composite rule find the derivative of the function y = log (sin (log *x*))

$$y' = \frac{1}{\sin(\log x)} \times \cos(\log x) \times \frac{1}{x} \quad //$$

The distance covered by a body in t seconds is given by $S = 4t - 5t^2 + 2t^3$, find the velocity and acceleration when t = 2 sec. **5**

OR

$$S' = \underline{4 - 10t + 6t^2}$$

$$v = S' = 4 - 10(2) + 6(2)^2 = \underline{4 - 20 + 24} = 8$$

$$a = S'' = 0 - 10(1) + 12t = -10 + 12t$$

$$a = -10 + 12(2) = 14 \text{ //}$$

OR

Distance travelled by a car is given by S = 160t – 16t^2 metre and time in seconds. When does the car stop ?

$$S' = 160(1) - 32t$$

$$\Rightarrow \quad V = 0, \quad \text{Car stop}$$

$$\Rightarrow \quad 32t = 160$$

$$t = \frac{160}{32} = 5$$

Find the maximum and minimum values of the function $x^3 - x^2 - x = 0$. **5**

OR

$$\Rightarrow \quad y = x^3 - x^2 - x$$

$$y' = 3x^2 - 2x - 1$$

$$y'' = 6x - 2$$

$$y' = 0$$

$$\Rightarrow \quad 3x^2 - 2x - 1 = 0$$

$$\boxed{x_1 = +1, \quad x_2 = -\frac{1}{3}}$$

$$y'' = 6(1) - 2$$

$$= 4 > 0$$

$$y = (1)^3 - (1)^2 - 1 = -1$$

$$=$$

$$y'' = \cancel{6}^{\,2}\left(-\frac{1}{\cancel{3}}\right) - 2 = -4$$

$$< 0$$

$$y = \left(-\frac{1}{3}\right)^3 - \left(-\frac{1}{3}\right)^2 - 1$$

$$= \frac{5}{27},$$

OR

Find the equation of the tangent to the curve $y = 2 - 3x + x^2$ at $(1, 2)$

$$y' = 0 - 3 + 2x$$

$$m = -3 + 2(1)$$

$$\boxed{m = -3 + 2 = -1}$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -1(x - 1)$$

$$y - 2 = -x + 1$$

$$\boxed{x + y - 3 = 0}$$

OR

Find the equation of the tangent to the curve $y = 2 - 3x + x^2$ at $(1, 2)$

(a) **Integrate :** $\cos x + e^x + \frac{1}{x} + x^2$, w.r. to x.

$$I = \sin x + e^x + \log x + \frac{x^3}{3} + C$$

The area under the curve $y = x^2$ between $x = 1$, and $x = 2$ is equal to …

$$A = \int_a^b y\,dx$$

$$= \int_1^2 x^2\,dx = \left(\frac{x^3}{3}\right)_1^2 = \frac{1}{3}\left(2^3 - 1^3\right)$$

$$= \frac{7}{3}$$ //

Using the rule of integration by parts evaluate the integral $\int x \sin 2x . dx$

ILATE

$$\int uv\,dx = uv_1 - \int v_1 u'\,dx$$

$u = x \qquad v = \sin 2x$

$u' = 1 \qquad v_1 = -\frac{\cos 2x}{2}$

$$= x\left(-\frac{\cos 2x}{2}\right) - \int \left(-\frac{\cos 2x}{2}\right)(1)\,dx$$

$$= -x\frac{\cos 2x}{2} + \frac{1}{2} \times \frac{\sin 2x}{2} + C$$

Evaluate $\int \sin 2x \cos 3x \, dx$

$$\sin A \cos B = \frac{1}{2}\left[\left(\sin(5x) + \sin(-x)\right)\right]$$

$$= \frac{1}{2}\left[\sin 5x - \sin x\right]$$

$$\int \sin 2x \cos 3x \, dx = \frac{1}{2}\int (\sin 5x - \sin x)\, dx$$

$$= \frac{1}{2}\left[\frac{-\cos 5x}{5} + \cos x\right] + c$$

Find $\int_0^{\pi/2} \sin^2 x . dx$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$I = \int_0^{\pi/2} \left(\frac{1 - \cos 2x}{2} \right) dx$$

$$= \frac{1}{2} \left[x - \frac{\sin 2x}{2} \right]_0^{\pi/2}$$

$$= \frac{1}{2} \left[\left(\frac{\pi}{2} - 0 \right) - \cancel{\frac{1}{2} \left(\sin \pi - \sin 0 \right)} \right]$$

$$= \frac{\pi}{4} \text{ //}$$

The area enclosed by the curve y = $x^2 + 1$, x-axis between $x = 1$, $x = 3$, calculate the area enclosed.

$$A = \int_a^b y\,dx$$

$$= \int_1^3 (x^2+1)\,dx = \left(\frac{x^3}{3} + x\right)_1^3$$

$$= \frac{1}{3}(3^3 - 1^3) + (3-1)$$

$$= \frac{1}{3}(27 - 1) + 2$$

$$= \frac{26}{3} + 2 = \frac{32}{3}$$

OR

Find the volume generated by rotating the curve $y = \sqrt{x+2}$ about x-axis between $x = 0$ and $x = 2$.

$$V = \int_a^b \pi y^2 \, dx$$

$$= \pi \int_0^2 \left(\sqrt{x+2}\right)^2 dx$$

$$= \pi \left[\frac{x^2}{2} + 2x \right]_0^2$$

$$= \pi \left[\frac{4}{2} + 4 \right] = \frac{12\pi}{2} \quad \boxed{= 6\pi}$$